Esotericism and Mathematics

John F. Nash

It seems to me now that mathematics is capable of an artistic excellence as great as that of any music, perhaps greater; not because the pleasure it gives . . . but because it gives in absolute perfection that combination, characteristic of great art, of godlike freedom, with the sense of inevitable destiny; because, in fact, it constructs an ideal world where everything is perfect and yet true. (Mathematician and philosopher Bertrand Russell[1])

Summary

Mathematics is a system of thought whose applications range from simple arithmetic, to modeling physical phenomena, to exploring abstract concepts that transcend the world we know. It is also a universal language that can express esoteric truths: perhaps a derivative of the language Sensa used by high initiates. Even mathematicians who disavow "the sacred" may experience a sense of awe upon gaining new insights or witnessing new discoveries.

This article reviews areas of exoteric and esoteric mathematics in order to build a more meaningful synthesis. A major focus is on the geometry, algebra, and esoteric symbolism of the point, the line, and the circle—along with three enigmatic numbers that feature therein. The article continues with an exploration of chaos theory, placing it in the context of the eternal conflict between cosmic order and disorder, stasis and evolution. The article concludes by reflecting on the larger role of mathematics in esotericism, and its actual and potential relevance to the spiritual path.

Introduction

Galileo Galilei (1564–1642) proclaimed: "The universe cannot be read until we have learned the language and become familiar with the characters in which it is written. It is written in mathematical language, and the letters are triangles, circles and other geometrical figures, without which . . . it is humanly impossible to comprehend a single word. Without these, one is wandering about in a dark labyrinth."[2] Most likely, "the universe" he was referring to was the external one, which his telescope was bringing into focus, and his statement pertained to applied mathematics. But it could equally pertain to pure mathematics and the universe of abstract mental formulations.

In either case, Galileo was expressing an appreciation, shared by many others, of the power and the *necessity* of mathematics. Its name comes from the Greek *mathema*, which means "knowledge," "study" or "learning." Clearly, it is a broad field, whose roots lie deep in antiquity, but which is more relevant today than ever before.

From early times, mathematics has been recognized as possessing esoteric as well as exoteric dimensions. The esoteric dimensions may have been paramount during Lemurian and Atlantean times, when initiates formed the priesthood. The language of adepts, we are told, was and remains Sensa: "a universal language, and largely a hieroglyphic cypher";[3] an "ideographic language."[4]

About the Author

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In his study of the ancient language, Dorje Jinpa explained that, at the highest levels of reality, Sensa consists of ideograms of enormous power. But as information filters down to lower levels—as the formless descends into form—the familiar process of differentiation and multiplication takes place. By the time the information reaches the physical level, many words, sounds, symbols and gestures may be needed to capture the meaning and power of the original ideograms. Among the relevant symbols are the circle, the point, and the line, discussed in some detail in the present article. Jinpa urged: "[T]he understanding of symbols must be intuitive rather than analytical [because] esoteric symbols represent ideas on a level where the parts, the dualities of opposites, are realized as a synthesis."[5] Yet Jinpa did not denigrate analysis; in particular, he paid tribute to Pythagoras (c.570–c.495 BCE) for his work on the concept of number.[6]

The esoteric literature is replete with articles and books on "sacred mathematics," divided into topic areas that include numerology, gematria, and sacred geometry. Much of the material lacks a sense of closure: fascinating but leaving us wondering what it all means. The extant material may well be a dim memory of the mathematical content of Sensa, used by the Masters before they withdrew from external manifestation at the end of the Atlantean era. The methods survived, but much of the underlying message was lost.

Sacred mathematics has largely ignored advances in its exoteric counterpart over the last 1,000 years.[7] Yet the dim memories are giving way to new insights, raising hope that mathematics will once again be recognized as a sub-language of Sensa.

Esotericist Alice Bailey (1880–1949) wrote of a geometric language that expresses our progress on the Path of Return:

> Each life that the Personality leads is, at the close, represented by some geometrical figure, some utilization of the lines of the cube, and their demonstration in a form of some kind. Intricate and uncertain in outline and crude in design are the forms of the earlier lives; definite and clear in outline are the forms built by the average advanced man of this generation. But when he steps upon the Path of Discipleship, the purpose consists in merging all these many lines into one line, and gradually this consummation is achieved. The Master is He Who has blended all the lines of fivefold development first into the three, and then into the one. The six-pointed star becomes the five-pointed star, the cube becomes the triangle, and the triangle becomes the one; whilst the one (at the end of the greater cycle) becomes the point in the circle of manifestation.[8]

Elsewhere Bailey spoke of the Great Architect, the Great Geometrician, who produces the blueprints "with their mathematical accuracy, their structural unity and their geometrical perfection."[9] She also identified exactitude in thought, higher mathematics, and philosophy as "methods of development" for those on the Third Ray.[10]

This article does not simply *discuss* mathematics, it offers readers the opportunity to *experience* mathematics, and to place it in relation to esotericism by exploring formal logic, analytical methods, applications, and formats for expressing mathematical solutions.[11] The reader will encounter numbers, symbols, equations and graphs. This is not the first esoteric text to include such material. In 1989, William Eisen published a series of lectures with mathematical content, some similar to that presented herein.[12] Eisen drew upon Rosicrucian teachings and "the Cabalah," as the basis for his interpretations. Unfortunately, many of those interpretations lacked persuasive force, and the work evoked little response. The present article looks to the trans-Himalayan teachings as its primary source of esoteric knowledge.

Notwithstanding Eisen's work, the mathematics presented here is more advanced than that found in earlier treatments of the subject. Some readers may welcome the innovation, while others may react negatively. Despite its long tradition and increasing importance, mathematics is not a popular subject in our culture; too often we celebrate ignorance rather than expertise.[13] This attitude must change if we are to move forward.

To understand the mathematics in this article, in a detailed sense, would be rewarding. But it is not necessary in order to grasp the article's intended message. Just as one need not understand Latin or Old Church Slavonic to appreciate the sanctity of the Mass, the esoteric aspects of mathematics can be appreciated without the need to verify every equation. The mathematics should stimulate the intuition; its elegance should be recognized; it should be regarded as a basis for contemplation.

The present article provides a very brief account of the development of mathematics, as an academic and utilitarian discipline, drawing attention both to its enormous success and to the recent discovery of inherent weaknesses. It illustrates traditional "sacred mathematics" by briefly reviewing gematria and the sacred geometry of the *vesica piscis*. The article's main purpose is to explore areas of mathematics, and their relevant analytical methods, that have not previously been addressed in the esoteric literature. It seeks to expand the boundaries of what may be considered esoteric mathematics and to simulate work on similar lines by other authors.

A major focus is on the geometric symbols of the point, the line, and the circle—forms that not only have an impressive array of numerical and algebraic properties, but also have profound esoteric significance. The mathematical constant, π (*pi*), which links the line and the circle, is a quantity that has been studied throughout the ages—and continues to captivate the imagination of the masses as well as of mathematicians. The article also examines the relatively new field of chaos theory, which forced science to abandon trust in calculability and to redefine the concept of determinism. Computations of the development of chaotic systems can provide patterns of exquisite beauty, worthy to inspire meditation.

The article ends with an effort to synthesize the various topics and to identify connections among diverse areas of exoteric and esoteric mathematics. It also reflects on the role of mathematics as a sacred language—potentially an expanding role as we come closer to recapturing some of the richness of Sensa on our individual and collective Path of Return. The study of mathematics is to be recognized as a spiritual path in its own right—with inherent opportunities and challenges.

A new root race will soon emerge on this globe, which will place less emphasis on the intellect than the fifth root race has done. Before long, mathematics may be viewed in a new light, and may assume new roles, as the intuitive faculties become firmly established in human consciousness.

Development of Mathematics

The origins of mathematics go back far in history, even prehistory. Every significant culture—Indian, Chinese, Babylonian, Egyptian, Greek, Mayan—encouraged the study of mathematics. The need to count and measure traded and warehoused goods laid the foundations of arithmetic. Land surveying stimulated the study of geometry; the very term is derived from the Greek *geometria*, which means "measuring the Earth." But even in antiquity mathematics was seen as an art transcending the purely utilitarian.

The Vedas of the Indus Valley of India, the world's oldest known scriptures, attached Sanskrit names to numbers extending over an enormous range of magnitudes. For example, one *mahayuga* equaled 10^{62}—a number one hundred times greater than the ratio of the diameter of the observable universe to the Planck length (the shortest distance that has any meaning in quantum mechanics). An impressive array of arithmetic, algebraic and geometric operations—some outperforming modern western counterparts in their simplicity—has been gleaned from the Vedas.[14]

The Greeks made substantial contributions to mathematics. Pythagoras studied the applications of arithmetic and discovered a synthesis of number, geometry, music and astronomy. Geometry was studied by Thales (c.624–c.546 BCE), and in the fourth-century BCE Euclid developed most of the theorems of high-school geometry we know today. Aristotle proposed elements of formal logic still used today as basic tools in pure mathematics.

Rhetorical algebra, in which unknown quantities were designated by words, can be traced back to Babylonian sources. Another example is found in Plato's *Timaeus*, where a passage can be interpreted as a pair of simultaneous algebraic equations, which can be solved to form a useful result.[15] Later, the seventh-century CE Indian mathematician Brahmagupta referred to the unknowns by colors.

What we now know as the Hindu-Arabic numbering system—the digits 1 through 9—probably developed in India around the beginning of the Common Era. Brahmagupta may have been the first to propose rules for incorporating zero into arithmetic calculations.

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The medieval period was the golden age of Islamic mathematics. In the ninth-century CE the Persian mathematician Muḥammad ibn Musa al-Khwarizmi translated Indian works on arithmetic and algebra into Arabic. He also wrote a treatise on trigonometry, which studies the relationship between the proportions and angles of triangles. The thirteenth-century Moroccan Sufi and mathematician Ibn al Banna al Marrakushi al-Azdi laid the foundations of modern *symbolic algebra*, in which unknowns are represented by symbols like x and y.

The work of the Islamic mathematicians was soon brought to the West, along with the Hindu-Arabic notation and the decimal system for expressing fractions. From the Renaissance onward the focus of mathematical study moved to western Europe. The differential calculus, which refuted Zeno's Arrow Paradox, was developed in the seventeenth-century CE by Isaac Newton and Gottfried von Leibniz.[16] The last several centuries have seen a great expansion of mathematics, and brilliant mathematicians have come from every part of the world.

Mathematics Reigns Supreme

Mathematics, as the term is used today, includes the study of quantity, pattern, space, and change. *Pure mathematics* long ago branched out beyond the world of objective reality. For example, "quantity" includes imaginary numbers, and Hilbert space can have an infinite number of dimensions. In many cases, however, topics in pure mathematics, once considered irrelevant to the real world, turned out to have important practical applications.

The renowned German mathematician Carl Friedrich Gauss (1777–1855) was among the first to give his field the accolade "Queen of the Sciences";[17] many others have done so since. Alone among the sciences, pure mathematics did not depend for its validity on laboratory or astronomical observation; it was the domain of pure thought, unsullied by the messiness of empiricism.

A major objective of pure mathematics is the derivation of *theorems* from stated premises, or *axioms*, using rules of logical inference. A theorem is a mathematical statement that can be tested to determine whether it is true or false. A famous example is Fermat's Last Theorem, proposed by the French mathematician Pierre de Fermat in 1637, but not proven until 1995. It states that no three positive integers a, b, and c can satisfy the equation $a^n + b^n = c^n$ for any integer value of n greater than two. Some theorems are derived "from first principles." More commonly, they invoke theorems already proven, in order to reduce the number of logical steps required. New theorems build upon existing ones.

At one time, logical operations were expressed in the language of the particular nation or culture: Latin, German, English, as the case might be. More recently, universal languages with their own alphabets, vocabulary and grammars have been developed to avoid ambiguities and misinterpretations arising from translation.

An early development, dating from 1847, was Boolean Algebra, named for the British logician George Boole. The variables in Boolean Algebra can take one of two values: true, conventionally designated by 1; and false, designated by 0. Three basic operations are: AND (represented by the symbol $\wedge$), "inclusive" OR ($\vee$), and NOT ($\neg$). To illustrate, $x \wedge y = 1$ means that both variables are true, while $x \vee y = 1$ means that at least one variable is true. The expression $\neg x = 1$, implies that x is false. Arithmetic operations can be performed on the variables; for example:[18]

$$x \wedge y = x \times y$$
$$\neg x = 1 - x.$$

On a much larger scale, the monumental *Principia Mathematica*, published by Alfred North Whitehead and Bertrand Russell in 1910–1913, attempted to express all rules of logical inference in algebraic form. It employed symbols like ∀ ("for all"), ∃ ("there exists"), and ⊂ ("is a subset of"). A second objective of *Principia Mathematica* was to identify a set of axioms sufficient for the whole of mathematics.

Axioms are definitions or assertions that establish the "rules of the game"—that is, the particular game being played.[19] Examples are: "n is a positive number"; "$a + b = b + a$, $a \times b = b \times a$" (commutability); and "Given infinitely many non-empty sets, one element can be chosen from each of those sets." Most theorems depend on multiple axioms, which must be mutually consistent and complete if a theorem is to be proved or disproved. Axioms are consistent if one does not contradict another; they are complete if the truth or falsehood of all dependent theorems can be decided. A desirable, but not essential, quality is that the axioms contain no redundancies.

Pure mathematics, which concerns itself with abstract concepts, occupies a relatively small number of people, mostly in academia. By contrast, millions of people are engaged in *applied mathematics* whose objective is to model real-world phenomena, make useful predictions, and keep track of the innumerable metrics of modern life. Whereas the theorems of pure mathematics are validated by logic, the models of applied mathematics are validated by comparison with relevant data. If the data support a model's predictions, it is used—hopefully with due caution—until defects cast doubts on its continued utility, and motivation grows to develop a better model.

Mathematics is applied to a vast range of problems in the physical sciences, technology, medicine, the social sciences, business, government, the military, and many other fields. Computer technology has enabled the solution of mathematical problems that would otherwise be impossible. Images, audio and video recordings, and data transmission are digitized and can be manipulated mathematically. The human genome has been sequenced, identifying some three billion base pairs constructed from a four-character alphabet. Internet search engines depend on mathematical algorithms to identify websites of potential interest or relevance.

Computer technology also made good use of mathematics. For example, Boolean algebra, with its truth values 1 and 0, is directly applicable to digital processing, in which the presence or absence of a voltage in binary circuitry (or the magnetic polarity on a hard drive, electric charge on a flash drive, or pit on an optical disk) encodes the numbers 1 and 0.

Nowhere has mathematics been applied more successfully than in the "hard sciences" of physics, chemistry and astronomy. Isaac Newton (1642–1727) demonstrated that the laws of motion and gravitation applied both to terrestrial phenomena and to planetary orbits—a shocking revelation to those who believed that celestial bodies were moved by the hand of God. In the nineteenth-century James Maxwell derived the equations of electromagnetic radiation, demonstrating that electricity, magnetism, and light are manifestations of a single phenomenon.

In 1905, Albert Einstein proposed the Special Theory of Relativity, which included the most famous equation of all time: $E = mc^2$, where E is energy, m is mass, and c is the speed of light. Bringing together three of the most fundamental ingredients of physical reality was an achievement of profound synthesis. The constant of proportionality, c^2, between energy and mass is so large—on the order of 10^{17} $(km/sec)^2$—that a small amount of mass can be converted into an enormous amount of energy. That conversion is demonstrated in nuclear reactors and, to deadly effect, in nuclear weapons.

Ten years later Einstein published his General Theory of Relativity, which utilized work on non- Euclidean geometry previously thought to be a fanciful construction of pure mathematics. The General Theory is far-reaching in its implications, but mathematical notation is so concise that its primary component, the Field Equation, can be expressed as: $G = (8\pi G/c^4) T$, where G is the gravitational constant and c is the speed of light. G and T are *tensors*, the higher-order equivalents of physical variables: G expresses the curvature of space-time, and T expresses energy or mass.[20] The General Theory of Relativity asserts that a massive object warps space-time, while, in return, the curvature of space-time determines the object's motion. Astronomical observations support the theory, and no new theory has yet challenged it.

In quantum mechanics, which relates to physical matter on subatomic scales, the behavior of and interactions among elementary particles are modeled by mathematical equations, notably the Schrödinger Wave Equation, formulated in 1925 by Austrian physicist Erwin Schrödinger. Its predictions have been verified by an enormous body of experimental data, to the point where few scientists doubt its validity.

Yet the inferences from quantum mechanics defy interpretation, and in many cases are counterintuitive. For example, at the quantum level, time runs both forward and backward; a particle's location and momentum cannot be specified simultaneously; particles often behave like waves. Schrödinger's Wave Equation is deterministic, but its dependent variables are probabilities attached to a *range* of states (charge, spin, momentum, and so forth), super imposed upon one another.[21] A specific value is obtained only upon measurement or observation.[22] How these behavioral oddities give way, with increasing scale, to our familiar, everyday reality remains to be explained.

At the largest scales—relating to galaxies and so forth—the driving force behind activity and the effect of that activity, is *geometry*: the non-Euclidian geometry of four-dimensional space-time. At the very smallest scales the distinction between physical reality and the equations that model it begins to break down; the basic building blocks of physical matter may not just be *described by* mathematical equations, they may *be* equations.[23] The mathematization of reality would have been endorsed by the Pythagoreans, and possibly by the Kabbalists who labeled the cascading emanations from Deity *sephiroth* ("numbers"). Their endorsement would not suggest a desire to drag reality down to the mundane level; rather, it would affirm that mathematics lifts what we experience as physical reality to the divine level. In 1937 British mathematical physicist James Jeans famously declared:

> Today there is a wide measure of agreement, which on the physical side of science approaches almost to unanimity, that the stream of knowledge is heading towards a non-mechanical reality; the universe begins to look more like a great thought than like a great machine. Mind no longer appears as an accidental intruder into the realm of matter; we are beginning to suspect that we ought rather to hail it as a creator and governor of the realm of matter.[24]

This realization came as no surprise to students of South Asian religions and philosophies, but it was shocking to the western mind, so strongly attached to notions of objective physical reality—or perhaps, we should say, so deeply mired in *maya*. Pure mathematics is a thought experiment; applied mathematics may prove to be the same.

The goal of unifying all the forces of nature—gravity, electromagnetism, and the strong and weak forces that operate at subatomic levels—currently remains unrealized. But many scientists are working on the problem, and the longsought-after "Theory of Everything" may emerge within the foreseeable future.

Cracks in the Walls

Over a period of centuries pure mathematics grew into a vast edifice of axioms, theorems and proofs. In order to prove a theorem's truth or falsity, one must be confident that axioms are complete and consistent, and until the early twentieth century it was generally believed that consistency and completeness could always be tested. If inconsistencies came to light, the problem could be resolved by restating the axioms, and incompleteness could be resolved by adding more axioms. "Mathematics," asserted a popular claim, "is the only exact science; the only [one] whose propositions are capable of conclusive proof and demonstration."[25]

Yet even in antiquity statements were constructed that could be considered neither true nor false. Epimenides (c. 600 BCE), a native of Crete, famously declared: "All Cretans are liars." To call that statement either true or false triggers an automatic and immediate rebuttal. The problem arises from self-reference. It may reside in the axioms, logical inferences, or both; and it may not be as obvious as in Epimenides's Paradox.

In 1931 the Austrian-born logician Kurt Gödel (1906–1978) demonstrated that consistency and completeness are mutually incompatible and inherently indeterminable. Specifically, he declared: if an axiomatic system is consistent it cannot be complete, and the consistency of axioms cannot be proven within the system itself.[26]

Gödel's Incompleteness Theorem was demonstrated for the special case of the arithmetic of natural numbers (the numbers 1, 2, 3, 4, …). But it was immediately recognized as having broader implications. The ability to prove or disprove any given theorem was no longer assured. And the dream, expressed in *Principia Mathematica*, of identifying a universal set of axioms was dashed. Mathematics was still enormously powerful, but cracks had developed in the walls of what had always been regarded as an impregnable fortress of rationality. Cracks also developed in applied mathematics. An important example is chaos theory, which, as a field of serious study, dates from the 1960s. Chaotic phenomena will be examined later in this article.

Aside from the inherent limitations of mathematics, serious issues arise from its misapplication or misinterpretation. Models of economic, social, political and biological systems inevitably simplify the real-world situation to which they apply, incorporating only those variables believed to be important, or those which can be measured. Inevitably, they are at the mercy of *exogenous* factors—those omitted from the models—which may have large, unforeseen influences.[27]

Statistical methods are used to analyze large volumes of historical data. They have afforded scientific legitimacy to a number of fields, including psychology and public health, where hypotheses might otherwise rest on anecdotal data or rumor. On the other hand, predictions must be treated with caution. For example: "Candidate A leads B by 10 points, going into the election, with a margin of error of 5 points" (a very dubious caveat); "California can expect a major earthquake in the next thirty years"; "the three-year survival rate for a patient with your condition is no more than 5 percent." Statistical predictions are probabilities, but the one future event that really matters may lie far from the expected value. Furthermore, the "tails" of probability distributions—percentiles less than five or greater than ninety-five—are notoriously inaccurate. Significantly, the tails are much wider in chaotic phenomena than in the Gaussian distributions typically used in statistical analyses.

Traditional Sacred Mathematics

The development of mathematics was stimulated by the need to count and measure, but numbers and geometric shapes were also believed to have larger meaning. Pythagoras' work on numbers has already been mentioned. Mathematics was a sacred science, taught in the ancient mystery schools. A significant overlap between mundane and sacred mathematics survived until about the sixteenth century.

Eventually, the two branches of divided, and exoteric mathematics experienced the impressive development described in the previous section. Sacred mathematics lived on and experienced a renewal of interest in the nineteenth and twentieth centuries.[28] But it virtually ignored developments in pure and applied mathematics. Most of the techniques of arithmetic and geometry employed in sacred mathematics were known in ancient Greece.

Two areas of sacred mathematics that have received much attention are gematria—along with its close cousin, numerology—and sacred geometry. In some instances, unifying links are identified between them.

Gematria and Numerology

Gematria, also known as *theomatics*, had its origins in ancient languages in which symbols denoted both letters and numbers. Letter–number equivalency allowed the numerical values of complete words or phrases to be calculated, compared with the values of other words or phrases, and studied for hidden meanings.

Hebrew is the sacred language of Judaism: the media of scripture and liturgy. Jews of the biblical period believed that every letter of the Hebrew alphabet was a divine revelation; the alphabet was to them what numbers were to Pythagoras. The Hebrew gematria extends back three millennia, but it gained particular favor in the medieval Kabbalah and in the Hermeticism of more recent times. The Greek gematria was known in the third century BCE, and significant inscriptions were found at Pompeii. Greek became the sacred language of early Christianity and continues to play that role in the Eastern Orthodox churches.[29] The nineteenth century saw a revival of interest in Greek gematria, though words and phrases of classical and Christian Greece remained its primary focus. The numbers associated with Hebrew and Greek letters are shown in Table 1.[30]

With numbers assigned to each letter of the alphabet, the gematric values of whole words can be found by summing the values of the individual letters. To illustrate, in Hebrew gematria, "Adam," אדם, has a value of 1 + 4 + 40 = 45.[31] The Greek word for God, Θεος, has a value of 9 + 5 + 70 + 200 = 284. Words of the same numerical value are believed to have a connection or mutual resonance. For example, the Hebrew words for Moon, ירח, and Arcana ("Mystery"), רזיא, both have the value 218, while Serpent, נחש, and Messiah, משיח, share the value 358. In Greek gematria, Sacred, Αγιως, and Jerusalem, Ιερουσαλημ, share the value 864. Brave souls look for resonances that cross linguistic boundaries, attaching significance to words or phrases that have the same numerical value in Hebrew, Greek, or one of the several versions of Latin or English gematria.[32]

Much has been made of the observation that "Jesus," Ιησους, has the value of 888 in Greek gematria. The number 888—in common with 111, 222, 333, 444, 555, 666, 777 and 999—is divisible by thirty-seven; 888 = 24 × 37. The Greek word for Christ, Χριστος, has a gematric value of 1,480, which is also divisible by thirty-seven: 1,480 = 40 × 37. Frederick Bligh Bond and Thomas Lea identified more than 100 words and phrases related to Jesus Christ whose values are multiples of thirty-seven.[33] Not surprisingly, the value of "Jesus Christ," Ιησους Χριστος, itself is so divisible: 2,368 = 64 × 37. Those who seek evidence of the sacred destiny of the United States note that 888 × 2 = 1776, the date of the Declaration of Independence.[34]

Gematria is not the only way that high entities acquire numerical values comprising repeated digits. Alice Bailey declared that 777 is the number of our Planetary Logos, "just as 666 and 888 holds the mystery hid of two other Heavenly Men. This number 777 is also the number of transmutation, which is the fundamental work of all the Heavenly Men."[35] She also stated that 666 "is the number of the active intelligent man and distinguishes his form nature from his spiritual nature, which is 999."[36] In *Revelation*, 666 is said to be the "number of the Beast,"[37] but in Hebrew gematria it is the number of Sorath, סורת, the Spirit of the Sun.[38]

Table 1. Equivalence Between Letters and Numbers

(a) Hebrew Gematria

Hebrew Letter		Trans-literation	Value
aleph	א	A	1
beth	ב	B	2
gimel	ג	G	3
daleth	ד	D	4
he	ה	H	5
vau	ו	V	6
zayin	ז	Z	7
cheth	ח	Ch	8
teth	ט	T	9
yod	י	Y	10
kaph	כ, ך	K	20
lamed	ל	L	30
mem	מ, ם	M	40
nun	נ, ן	N	50
samekh	ס	S	60
ayin	ע	O	70
pé	פ, ף	P	80
tzaddi	צ, ץ	Tz	90
qoph	ק	Q	100
resh	ר	R	200
shin	ש	Sh	300
tau	ת	Th	400

(b) Greek Gematria

Greek Letter		Trans-literation	Value
alpha	α, Α	a, A	1
beta	β, Β	b, B	2
gamma	γ, Γ	g, G	3
delta	δ, Δ	d, D	4
epsilon	ε, Ε	e, E	5
zeta	ζ, Ζ	z, Z	7
eta	η, Η	h, H	8
theta	θ, Θ	th, Th	9
iota	ι, Ι	i, I	10
kappa	κ, Κ	k, K	20
lambda	λ, Λ	l, L	30
mu	μ, Μ	m, M	40
nu	ν, Ν	n, N	50
xi	ξ, Ξ	xi, Xi	60
omicron	ο, Ο	o, O	70
pi	π, Π	p, P	80
rho	ρ, Ρ	r, R	100
sigma	σ (or ς), Σ	s, S	200
tau	τ, Τ	t, T	300
upsilon	υ, Υ	u, U	400
phi	ϕ, Φ	ph, Ph	500
chi	χ, Χ	ch, Ch	600
psi	ψ, Ψ	ps, Ps	700
omega	ω, Ω	ō, Ō	800

Successive addition of digits reduces the gematric value of a word to its *root value*. Permissible root values are the digits 1 through 9, plus the *master numbers* 11 and 22. For example, "Adam," whose value in Hebrew gematria is 45, has a root value of 9: 4 + 5 = 9. "Christ," whose value in Greek gematria is 1,480, has a root value of 4: 1 + 4 + 8 + 0 = 13; 1 + 3 = 4. Words with the same root values are believed to have mutual resonance, though not as strong a resonance as among words with the same gematric values. Reduction to root values links gematria with more traditional forms of numerology.

Gematria depends for its validity on a stable alphabet, consistent spelling, and agreed-upon rules governing the assignment of numbers to the alphabet. Those requirements were met tolerably well in classical Hebrew and Greek, but are met less well in other languages. Several forms of English gematria have emerged, including at least two applicable to modern English that differ in their number-assignment rules. The Reduced Alpha Number (RAN) system assigns values to letters as shown in Table 2.[39]

Table 2. RAN System of English Gematria

1	2	3	4	5	6	7	8	9
A	B	C	D	E	F	G	H	I
J	K	L	M	N	O	P	Q	R
S	T	U	V	W	X	Y	Z	

Bailey made use of the RAN system and attached meanings to certain root values:

> [T]he numerical value of the word "four" is the same in detail as that of the word

> "force," if you eliminate the number five. For humanity, it is the fifth energy which leads to the battlefield, the energy of the discriminating mind, and when that has been in due time used, controlled and transmuted, "only the four remains and force has gone." Note the detail of the numbering:
>
> F O R C E 6 9 3 5 [with a root value of] 11. Number of adept, using energy. F O U R 6 6 3 9 [root value] 6. The creator, unifying the subjective and the objective.
>
> It is apparent that force in the first group ends in separativeness, for five is the number of the mind and of man. Number nine, the number of initiation, is hidden midway in force, but the climaxing figures indicate activity and separation. In the second group of figures, activity precedes the nine of initiation, and that nine is the culmination.[40]

Elsewhere, Bailey applied the RAN system to the word Shamballa:

> S.H.A.M.B.A.L.L.A. or 1.8.1.4.2.1.3.3.1. This word equals the number 24 which in its turn equals 6. I would call your attention to the fact that the word has in it nine letters, and—as you know—nine is the number of initiation. . . . The number 6 is the number of form or of manifestation, which is the agent or medium through which this realization comes and by which the consciousness is unfolded. . . . Again, 6 being the number of the sixth ray, it is therefore the number of idealism and of that driving force which makes mankind move forward upon the path and in response to the vision and press upward towards the light.[41]

John Berges made extensive use of the English gematria in his analysis of the Great Invocation.[42] Berges used the original 1945 version of the Invocation, and the numerological relationships he identified are not preserved in gender-neutral versions which have been proposed since then.

Bailey attached considerable importance to the number 24, which occurred in the gematric value of "Jesus" and also emerged as an intermediate step in the calculation on the root value of "Shamballa":

> The number 24 is of deep interest, expressing as it does the double 12—the greater and the lesser zodiac. Just as the number 6 expresses space, so the number 24 expresses time, and is the key to the great cycle of manifestation. It is the clue to all cyclic appearance or incarnation. Its two figures define the method of evolution; 2 equals the quality of love-wisdom, working under the Law of Attraction and drawing man from one point of attainment to another; whilst 4 indicates the technique of conflict and the achieving of harmony through that conflict; 4 is also the number of the human hierarchy, and 2 is the number of the spiritual hierarchy.[43]

The number 6 expresses space because there are six directions: up, down, east, west, north and south. Bailey added: "Needless to say, there is much more to say anent these figures, but the above will suffice to show the satisfactory nature of esoteric numerology—not numerology as understood today."[44]

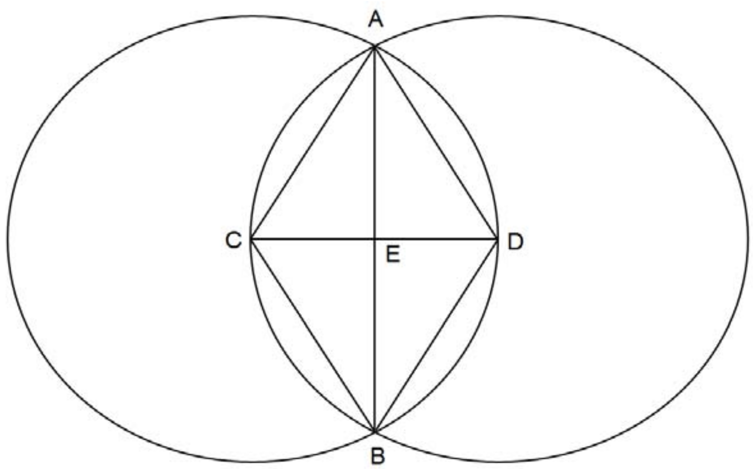
Traditional numerologists attach meaning to the numbers 1 through 9, 11 and 22.[45] A broad consensus exists that odd numbers: 1, 3, 5 … are "masculine," reaching out into new realms, while even numbers: 2, 4, 6 … are "feminine," restoring balance after each forward thrust. But a pervasive problem has always been the lack of agreement on numbers' more specific meanings. The trans-Himalayan teachings have created new meanings based on the seven planes of creation, the Seven Rays, and the twelve Creative Hierarchies;[46] the quotes cited above provided relevant examples.

Sacred Geometry

Sacred geometry is a broad field, ranging in its focus from simple geometric shapes, to structures like the Great Pyramid or Chartres Cathedral, to labyrinths and mazes, to large-scale features of the landscape like ley lines and the alignment of ancient monuments.[47] Within the sub-field of geometric shapes, a vast literature is available on the golden rectangle and its associations with the pentagram and the Fibonacci series;[48] they will not be discussed herein.

The brief illustration of sacred geometry, presented here, focuses on the geometric shape known as the *vesica piscis* (Latin: "fish bladder," or, more sensitively, "fish vessel"). Studied since antiquity, it is the area of overlap between two circles of equal radius, each centered on the circumference of the other (Figure 1). It is bounded by the two arcs ACB and ADB.

Figure 1. Vesica Piscis and Its Inscribed Triangles



The two interlacing circles have been interpreted as the realms of spirit and matter—though there is disagreement on which circle represents which realm.[49] The vesica piscis itself—the area where spirit and matter overlap—can be associated with the "cosmic soul."[50] The notion of the soul as the mediator between spirit and matter dates back to Plato:

> Out of the indivisible and unchangeable, and also out of that which is divisible and has to do with material bodies, [God] compounded a third and intermediate kind of essence [the soul], partaking of the nature of the same and of the other, and this compound he placed accordingly in a mean between the indivisible, and the divisible and material.[51]

Alice Bailey affirmed a similar relationship. The soul, she wrote, is "that entity which is brought into being when the spirit aspect and the matter aspect are related to each other."[52]

The vesica piscis was accorded special significance by the early Christians, and it is not surprising that they would have deemed the symbol of the fish important at the beginning of the Piscean Age. Since then, the vesica has been incorporated into artwork and numerous heraldic emblems, and possibly inspired the shape of the bishop's mitre and the architecture of the Gothic arch.

Two equilateral triangles can be inscribed within the vesica piscis: ACD and BCD. The two triangles have been associated with "the world above and the world below."[53] If one of the triangles is laid over the other, the hexagram, or Star of David, is produced. The triangle can symbolize many things, including the Trinity, the Rays of Aspect: Will or Power, Love-Wisdom, and Active Intelligence; body/personality, soul and spirit; and the Spiritual Triad.

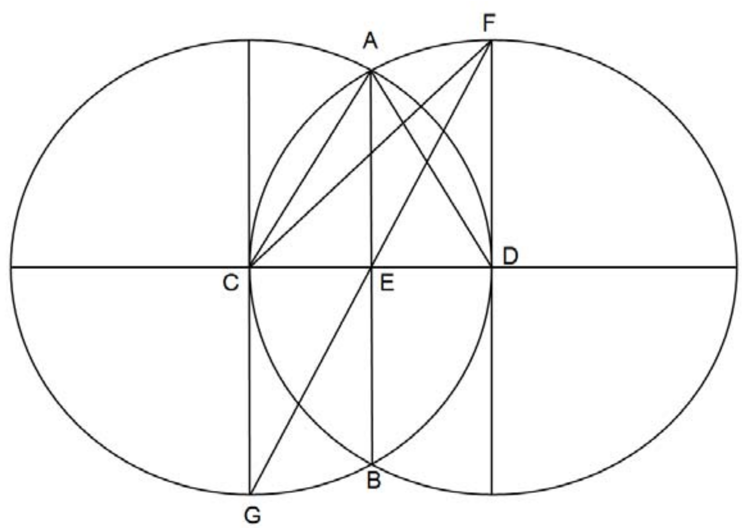
Several numerical relationships emerge from the vesica's proportions. The sides of the two inscribed equilateral triangles are equal to the radius of the circles. Together, the two triangles form a rhombus ACBD whose area is equal to the square of the radius. If the radius is 1 unit, then the height of the vesica and rhombus, AB, is equal to $\sqrt{3}$, or 1.7320508… Today, we recognize this as an *irrational* (literally, "not a ratio") number,[54] but the ancients devoted much effort to finding integer ratios that would equal or, at least, approximate it. In his third-century BCE treatise *On the Measurement of the Circle*, Archimedes of Syracuse, determined that the true value of √3 lay between 265/153 and 1,351/780, that is, between 1.73203... and 1.73205.... Their average is a very respectable 1.73204…

The nineteenth-century esotericist William Stirling used Greek gematria to relate the distances CD (= 1), AB (= √3), and the width of the whole figure (= 3) to the names of personages in Greek mythology. The numbers 1, √3, 3 are in approximately the same proportions as the gematric values of Hermes (Ερμης = 353), Zeus (Ζευς = 612), and Apollo (Απολλων = 1061).[55] It may be verified that 612/353 = 1.734…, and 1061/353 = 3.006…. The same result has also been expressed in the cryptic form: Zeus = √(Hermes × Apollo).[56] This equation is an example of rhetorical algebra.

Additional results for the vesica piscis are shown in Figure 2. The geometric construction provides a means to evaluate three useful square roots. In addition to AB (= √3), we have CF = √2 and FG = √5. The numbers 2, 3, and 5 belong to the Fibonacci series: 1, 2, 3, 5, 8 …, which has the property that each value in the series is the sum of the previous two.[57]

Finally, the angles generated in the figure are precisely the ones needed to create the polygons forming the faces of the five Platonic solids. The tetrahedron, octahedron and icosahedron are formed from equilateral triangles, of which ACD is an example. The cube is formed from squares, whose corners are right angles, like CDF. The dodecahedron is formed from regular pentagons whose interior angles are 108°, like CEF.[58]

Figure 2. Additional Results for the Vesica Piscis



The Point, the Line, and the Circle

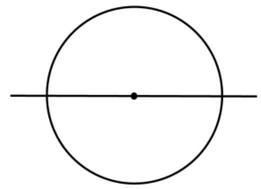
Figure 3. Celtic Cross. Christian cross superimposed on the disk of the sun.

Mathematical Properties

In antiquity people had three drafting instruments: a stylus, straightedge (or ruler), and compass. With one jab at the paper (papyrus, clay, slate, or other medium) they could produce a point. With a straightedge they could draw a line. And with a compass they could draw a circle. Inevitably, their drawing was not perfect. But Plato conjectured that the things of this world are just the imperfect shadows of perfect Forms residing in a higher world. Euclid applied the same principle to conceive of an idealized point, line and circle. The point had no dimensions; it was infinitely small. The line was infinitely thin (and straight). The circle was a closed curve of infinitesimal thickness and perfectly uniform radius.

Embedded within construction of the vesica piscis are three simpler geometric shapes: the point, the line, and the circle (Figure 4). In this section the rich mathematical properties of three shapes will be studied. Comments will also be made about three significant constants, or numbers, including π, which appear in the mathematics. The esoteric symbolism of the three geometric shapes will then be explored against the backdrop of the mathematical analysis.

Figure 4. The Point, the Line, and the Circle



So small is the idealized point, that the Russian-born mathematician Georg Cantor (1845–1918) proved that an *uncountably infinite* number of points can fit on a line segment of any length. By contrast ratios, like 2/3, 7/5 and 31/19, are infinite but *countable*.[59] Similarly, an uncountable number of points can fit on a circle of any radius. If, to quote William Blake, "infinity" can be held in the palm of one's hand, it can also be held in a short line, or the circumference of a small circle.

The diameter of a circle is twice the radius, r, and the circumference can be expressed either as π times the diameter, or as $2\pi r$. The mathematical constant, or number, π is equal to 3.14159265…. Methods of calculating it will be discussed later. Table 3 lists π and two other constants: e and i, that also play important roles in the present analysis. Euler's number, e, named for the eighteenth-century Swiss mathematician Leonard Euler, will be encountered later in this discussion. The imaginary unit, i, is the square-root of −1.

The first two constants: π and e are *real* numbers: they can be plotted on a line, given an origin and a suitable scale. They are also *irrational*: they cannot be expressed as the ratio of two integers. Furthermore, π and e are *transcendental*, which in this context means that they are not the roots of polynomial equations with rational coefficients.[60] By contrast $\sqrt{2}$ is irrational; but it is not transcendental, because it satisfies the equation $x^2 - 2 = 0$.

Table 3. Three Mathematical Constants

Symbol	Meaning	Value
π	Ratio of the circumference of a circle to its diameter	3.14159265…
e	"Euler's number." Base of natural logarithms, and base of the exponential function.	2.71828183…
i	Imaginary unit. Root of $x^2 + 1 = 0$.	√(-1)

The third number in Table 3, i, is neither irrational nor transcendental (it is the positive root of $x^2 + 1 = 0$); it is an *imaginary* integer.[61] It cannot be plotted on a line, yet it obeys the standard rules of arithmetic and can readily be incorporated in algebraic expressions. Other imaginary numbers can be formed by multiplying real numbers by i. Imaginary numbers were first recognized in the sixteenth century by Jeremy Cardan and Rafael Bombelli, but they were widely scorned, in much the same way as the Greeks scorned negative numbers. From Euler's time onward, mathematicians cheerfully accepted them.

In a Cartesian coordinate system, invented by René Descartes (1596–1650), a point is represented by a pair of values of x and y. A line is represented by an equation of the form: $y = ax + b$, where a is the slope of the line and b is its intercept on the y-axis. For convenience in analyzing our point, line and circle, we shall choose a coordinate system whose origin (0, 0) coincides with the point, and whose x-axis lies along the line (Figure 5). Thus the equation of the line reduces to $y = 0$.

The circle can be represented by the equation: $x^2 + y^2 = r^2$, where r is the radius. Alternatively, it can be represented in parametric form as: $x = r\cos\theta$, $y = r\sin\theta$, where cos and sin are the trigonometric cosine and sine functions (Figure 6). The variable θ is the angle—measured in a counterclockwise direction from the positive x axis—of the radius vector drawn from the origin to a point on the circle.

Figure 5. Circle in Cartesian Coordinates

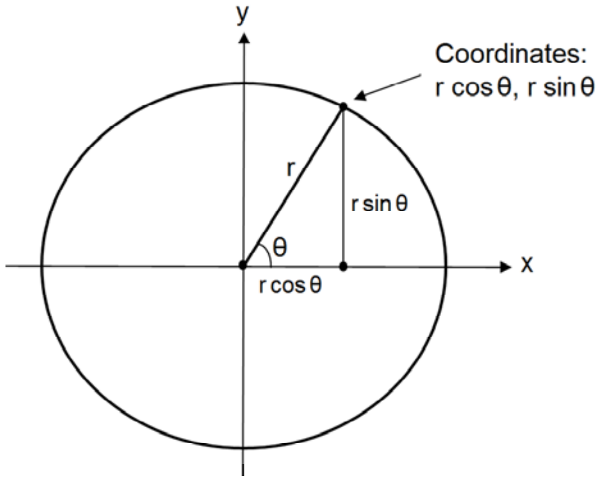


Figure 6. Sine and Cosine Functions

Sine (blue), Cosine (red)

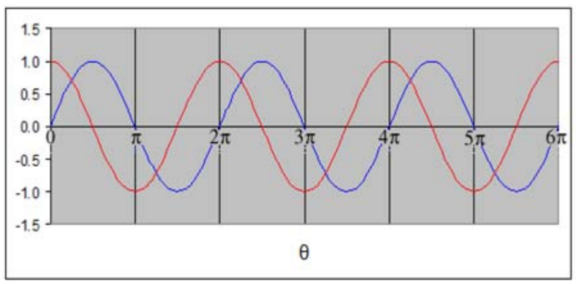
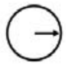
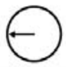


Figure 5 shows the situation where θ = 60°, or π/3 radians.[62] The coordinates of the point on the circle are r/2, √3r/2. Note that, by Pythagoras' theorem, the sum of the squares of the x and y coordinates equals the square of the radius of the circle; that is: $(r/2)^2 + (\sqrt{3}r/2)^2 = r^2$. As θ increases from zero to 2π (that is 360°), the radius vector sweeps through the whole area of the circle. The coordinates of the point on the circle, corresponding to θ = 0, π/2, π, 3π/2, and 2π, are listed in the third column of Table 4.[63] Rotation of the radius vector can continue indefinitely; θ increases by 2π with each complete rotation.

Table 4. Points on the Circle

θ	Radius vector	Cartesian (x, y) coordinates: $r\cos\theta$, $r\sin\theta$	Complex value: $re^{i\theta} = r(\cos\theta + i\sin\theta)$
0		r, 0	$re^{i0} = r$
$\pi/2$		0, r	$re^{i\pi/2} = ir$
π		– r, 0	$re^{i\pi} = -r$
$3\pi/2$		0, – r	$re^{3i\pi/2} = -ir$
2π		r, 0	$re^{2i\pi} = r$

An interesting variation on the Cartesian representation is obtained by plotting the circle in the *complex plane* (Figure 7).[64] The horizontal axis is now the *real axis*; and the vertical axis is the *imaginary axis*. On the latter, quantities are multiples or fractions of the imaginary unit, i. In the complex plane, a point is represented by a complex number, which is the sum of a real part and an imaginary part.[65] A point on the circle can be expressed in the form:

$$r(\cos\theta + i\sin\theta).$$

It can be shown that this expression is equivalent to the exponential function $re^{i\theta}$, where e is Euler's number, equal to 2.71828…, the second of the two irrational, transcendental numbers listed in Table 2.[66]

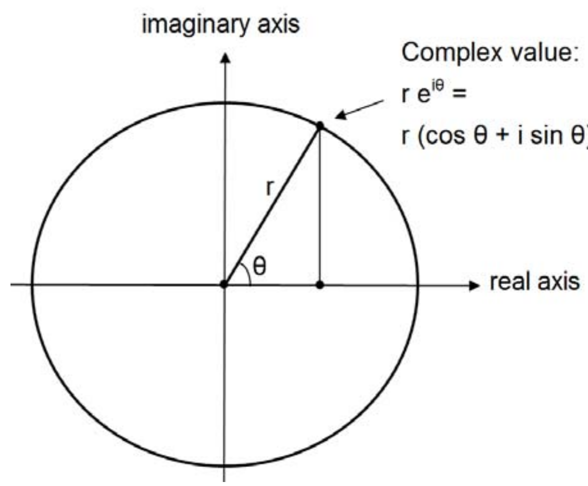
As before, Figure 7 shows the situation where $\theta = \pi/3$, or 60°. The point on the circle has the value:

$$re^{i\pi/3} = r\{\cos(\pi/3) + i\sin(\pi/3)\} =$$
$$r\{1/2 + (\sqrt{3}/2)i\} = r(1 + \sqrt{3}i)/2.$$

By Pythagoras' Theorem, the sum of the squares of the real and imaginary parts equals the square of the radius: $r^2\ (1 + 3)/4 = r^2$. Again, as the angle θ increases from zero to 2π, the radius vector sweeps through the area of the circle. Values of the point on the circle for $\theta = 0$, $\pi/2$, π, $3\pi/2$, and 2π are listed in the fourth column of Table 4.

The situation when $\theta = \pi$ calls for special attention. The radius vector lies along the negative real axis, and the point on the circle has the value $re^{i\pi} = r(\cos\pi - i\sin\pi) = -r$. From this result, we obtain the equation known as Euler's Identity: $e^{i\pi} + 1 = 0$. It incorporates the imaginary number i and the two most familiar irrational, transcendental numbers: π and e, expressing synthesis comparable to Einstein's $E = mc^2$. Euler's Identity has been described as "the most beautiful equation" and compared to a Shakespeare sonnet. The twentieth-century theoretical physicist Richard Feynman called it "our jewel" and "the most remarkable formula in mathematics."[67]

Figure 7. Circle in the Complex Plane



Ubiquitous π

The mathematical constant π traditionally was defined as the ratio of the circumference of a circle to its diameter (or twice the radius, r). The area of a circle is equal to πr^2. The surface area of a sphere is $4\pi r^2$—or exactly four times the area of its equatorial plane—and the volume of a sphere is $4\pi r^3/3$. In higher dimensions, π features in the "surface areas" and "volumes" of hyperspheres.

The Hebrew Bible assigned π a value of 3.[68] More serious attempts were made in ancient

Egypt and Greece to calculate the value of π. Closely related, were attempts to "square the circle": to construct a square with the same area as a circle, using nothing more than a straight-edge and compass. We now know that the latter attempts were futile.[69]

Good approximations to the value of π were obtained by measuring the perimeter of a regular polygon inscribed in, or circumscribed around, a circle. The greater the number of sides, the closer the polygon approximates the curvature of the circle, and the more accurate is the calculated value of π. For example, an inscribed square gives a value of $2\sqrt{2}$ = 2.828…, a superscribed square gives a value of 4, and their mean is 3.414… An inscribed hexagon gives a value of 3, a circumscribed hexagon gives a value of $2\sqrt{3}$ = 3.4641…, with a mean of 3.232…[70] In the work referred to earlier, Archimedes of Syracuse reportedly used polygons up to 96 sides and determined that $223/71 < \pi < 22/7$; that is, the true value of π lies between 3.1408… and 3.1429…, with a mean of 3.1419…. This was a laudable effort. Yet two centuries earlier, the Chinese mathematician Zu Chongzhi reportedly calculated π to seven decimal places.

In the seventeenth-century infinite series were discovered from which π could be calculated to any desired accuracy by taking a sufficient number of terms. Examples include an infinite sum developed by Leibniz, and an infinite product by John Wallis (1655):

$$\frac{\pi}{4} = \sum_{n=0}^{\infty}(-1)^n \frac{1}{2n+1}$$
$$= 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \frac{1}{13} - \frac{1}{15} + \ldots$$
$$\frac{\pi}{2} = \prod_{n=1}^{\infty}\left(\frac{2n}{2n-1}\right)\left(\frac{2n}{2n+1}\right) = \prod_{n=1}^{\infty}\frac{4n^2}{(4n^2-1)}$$
$$= \left(\frac{2}{1}\times\frac{2}{3}\right)\times\left(\frac{4}{3}\times\frac{4}{5}\right)\times\left(\frac{6}{5}\times\frac{6}{7}\right)\times\left(\frac{8}{7}\times\frac{8}{9}\right)\times\ldots$$
$$= \frac{4}{3} \times \frac{16}{15} \times \frac{36}{35} \times \frac{64}{63} \times \frac{100}{99} \times \frac{144}{143} \times \ldots$$

The relationship between π and the circle, juxtaposed against the linearity of the infinite series—using "linearity" in its everyday sense—affirms the connection between the symbolism of circle and line. Note that the infinite sum involves only odd numbers, which traditionally are regarded as masculine. The infinite product has even number—traditionally regarded as feminine—in the numerators and odd numbers in the denominators.

These infinite series have interesting and evocative structures, but they converge slowly; one thousand terms in the infinite sum and product yield an accuracy of only two decimal places. Much more efficient methods have recently been developed, including the Bailey–Borwein–Plouffe formula, published in 1995.[71] No more than forty decimal places are reckoned sufficient for any conceivable computation relevant to the physical universe; nevertheless, the calculation of more and more digits has become a challenge the world over. Today, π is known to more than 10^{13} digits. A consequence of its irrationality is that the decimal expansion of π never converges to a repetitive pattern; the sequence of digits is believed to be entirely random.[72]

Closely related to its role in the geometry of the circle, π defines the periodicity of the trigonometric functions. It also features in definite integrals that can be evaluated by the substitution of a trigonometric function.[73] More surprising, π appears in areas of mathematics remote from the geometry of the circle. It appeared in Einstein's Field Equation cited earlier. It occurs, along with the imaginary number i, in Schrödinger's Wave Equation. Moreover, π occurs in probability theory. The *normal* (Gaussian) probability density distribution, with mean μ and standard deviation σ, is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

The ubiquity of π across multiple areas of mathematics gives it greater significance than any other transcendental number enjoys. Not surprisingly, it has caught the public imagination; March 14, 2015 was celebrated as "Pi Day."[74]

Esoteric Symbolism

Throughout the ages, the circle, the line, and the point have been recognized as symbols of profound significance. They are represented in the Celtic cross, like the one shown at the beginning of this section (Figure 3). The Celtic cross is believed to be a superposition of the Christian cross on the disk of the sun.

In ancient cultures the circle often depicted the sun, the moon, or the sky, while the horizontal line depicted the earth or the horizon. The circle, bisected by the horizontal line (Figure 4), could represent the sun or moon rising above, or setting below, the horizon. Or it could symbolize the earth suspended between the dome of the firmament and the underworld. The fourteenth-century BCE Egyptian pharaoh Akhenaten incorporated the circle into sacred iconography to depict the solar deity. Modern esoteric teachings once more speak of a solar deity, whose threefold structure includes the "Physical Sun," the "Heart of the Sun," and the "Central Spiritual Sun."[75] Moreover, we understand that the etheric web of the Sun is patterned on "interlaced circles."[76]

The circle, with no beginning or end, represents the undifferentiated void, the primeval chaos: eternal and embracing the whole of space. The circle is the symbol of zero, the "non-number" from which the infinity of natural numbers proceeds. The rotating circle also represents the endless progression of time, day after day, year after year. Although we now know that planetary orbits are not perfectly circular, the sense of their endless rotation—and the confidence in the divine order it inspires—remains intact. The rotating circle also represents the Wheel of Rebirth, the "cyclic manifestation is the law of life," the cyclical nature of karma.

The three-dimensional analog of the circle is, of course, the sphere. The observation that God is "an infinite sphere, whose center is everywhere and whose circumference is nowhere" has been attributed variously to Hermes Trismegistus, the fourth-century CE philosopher Marius Victorinus, and the seventeenth-century mathematician Blaise Pascal. Many have found the observation evocative and meaningful.

The Neoplatonist Plotinus represented *Monas*, the highest aspect of his divine trinity, by the point within the circle. Significantly, it was the same as the hieroglyph of the Egyptian sun god Rā, and the same as the modern astrological symbol for the sun. Plotinus likened it to the stone, dropped into a pond that causes waves to radiate out across the surface of the water.[77]

In the trans-Himalayan teachings, the point within the circle represents potency, the origin of power, the source of life. In *The Secret Doctrine*, Helena Blavatsky quoted an "Archaic Manuscript" that interpreted the circle as "divine Unity, from which all proceeds, whither all returns."[78] Alice Bailey explained that, in the process of manifestation, the "will dynamic sweeps from the center to the periphery and builds the little world of form."[79] Four verses in the Great Invocation begin with a reference to either the point or the center: "From the point of Light within the Mind of God . . . . From the point of Love within the Heart of God . . . . From the center where the Will of God is known . . . . From the center which we call the race of men . . . .[80]

Again quoting from her ancient manuscript, Blavatsky identified the point within the circle as the first differentiation from the primeval Unity:

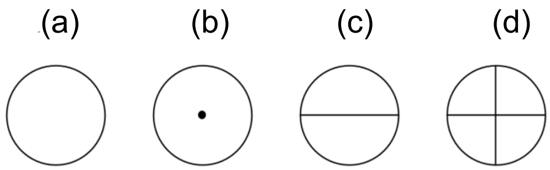
> The point in the hitherto immaculate Disk, Space and Eternity in Pralaya, denotes the dawn of differentiation. It is the Point in the Mundane Egg . . ., the germ within the latter which will become the Universe, the ALL, the boundless, periodical Kosmos, this germ being latent and active, periodically and by turns.[81]

Referring to four sketches (Figure 8), Blavatsky discussed the bisection of the circle by one diameter, and then by two orthogonal, diameters:

> The first illustration being a plain disc [(a)], the second one in the Archaic symbol shows [(b)] a disc with a point in it—the first differentiation in the periodical mani-

festations of the ever-eternal nature, sexless and infinite . . . the point in the disc, or potential Space within abstract Space. In its third stage the point is transformed into a diameter, thus [(c)]. It now symbolizes a divine immaculate Mother-Nature within the all-embracing absolute Infinitude. When the diameter line is crossed by a vertical one [(d)], it becomes the mundane cross.[82]

Figure 8. Circle, Central Point, and Diameters



Bailey listed the same symbols in her discussion of the initiatory secrets.[83] She also discussed in greater detail the circle, its central point, and its bisection by the line:

> 1. The circle. This stands for the ring-pass-not of undifferentiated matter. It stands for a solar system or the body logoic, viewed etherically; it stands for a planet or the body of a Heavenly Man viewed etherically; it stands for a human body, viewed likewise, etherically and it stands for them all at the prime or earliest epoch of manifestation. It stands finally for a single cell within the human vehicle, and for the atom of the chemist or physicist.
>
> 2. The circle with the point in the center. This signifies the production of heat in the heart of matter; the point of fire, the moment of the first rotary activity, the first straining of the atom, motivated by latent heat, into the sphere of influence of another atom. This produced the first radiation, the first pull of attraction, and the consequent setting up of a repulsion and therefore producing
>
> 3. The circle divided into two. This marks the active rotation and the beginning of the mobility of the atom of matter, and produces the subsequent extension of the influence of the positive point within the atom of matter till its sphere of influence extends from the center to the periphery. At the point where it touches the periphery it contacts the influence of the atoms in its environment; radiation is set up and the point of depression makes its appearance, marking the inflow and outflow of force or heat.[84]

At the macrocosmic level, the circle represents the ring-pass-not of the planetary, solar or cosmic entity, the domain created from the entity's own being and for which the entity is responsible. The Will aspect radiates concentric waves of potency from the center, like ripples on the surface of a pond. Manifestation spreads out to fill the ring-pass-not, only to be withdrawn once again into the point at the end of the manvantara. On a smaller scale, the circle symbolizes the ashram, with the master at the center, radiating his or her consciousness toward the periphery where disciples interact with the outer world. The ashram eventually completes its mission, and the master's consciousness is withdrawn to the center.

This process of exhalation and inhalation is repeated at all scales, including our own. Our lives, our periods of activity and introspection, our days and nights follow the same pattern. Bailey explained the change of direction that occurs when the aspirant moves onto the path of initiation:

> The aspirant must ever work from the outside to the within and must endeavor to direct his life from above downwards, if these forces are to be dominated by him and are not to control him. The initiate, however, works "from within the circle," that is the circle or field of maya. His activity must therefore be carried forward from the very heart of the mystery of these forces.[85]

The circle can symbolize the causal body, which, to quote Bailey, "will palpitate in due course of time with an inner irradiation, and an inner glowing flame that will gradually work its way from the centre to the periphery."[86] Bailey added that the flame "will then pierce through that periphery"; it will "mount upward to the Triad, and (becoming one with that Triad) will be re-absorbed into the spiritual consciousness."[87]

The line can represent the passage of time, the evolutionary process, and most importantly the Path of Return. The expansion of human consciousness initially is passive, and Blavatsky related the horizontal line to the feminine aspect ("the Mother-Nature"). But then the path becomes more active, and Will, a masculine quality, is symbolized by the vertical line. Addressing disciples on the First Ray of Will and Power, Bailey declared: "For you, there must be . . . a line."[88] For those on the sixth ray: "The path that you are on leads to the outer circle of the life of God; the line goes forward to the outer rim. Stand at the center."[89]

Given the observation that the physical universe is the manifestation of Thought, and mathematics' possible associations with Sensa, the language of high initiates, more esotericists should be encouraged to study mathematics. As they do so, they could become receptive to teachings that would be difficult or impossible for Higher Intelligences—on this planet or elsewhere—to communicate by other means.

The line can also symbolize *connection*: connection with the group and, when the antahkarana is built, with the higher mental subplanes. The vertical line is added to the horizontal, giving us the cross—a symbol whose roots go back far in history, but which became the supreme emblem of Christianity.

At the Fourth Initiation, the disciple ascends that cross to make the ultimate renunciation of the lower self.[90] The flame burns through the lower vehicles and the periphery of the causal body—though not through the monadic ring-pass-not. There remains "only the vertical line 'reaching from Heaven to Hell.'" From then on, the "goal of the initiate (between the fourth and the seventh initiations) is to resolve the line into the circle, and thus fulfill the law and the 'rounding out' of the evolutionary process."[91]

Analysis of the three geometric figures in both Cartesian coordinates and the complex plane yields additional insights. Cartesian coordinates adhere to a physical description of space, while the complex plane resides at a higher level of abstraction. Leibniz associated imaginary numbers, irreverently, with the Holy Spirit, and called the imaginary unit i "that amphibian between being and nonbeing."[92] Perhaps we could say that it hovers between the "seen and unseen," mentioned in the Nicene Creed.[93] Complex numbers overcome the duality of Cartesian coordinates, offering synthesis without destroying distinction.

The exponential function is a remarkable device. Normally, it models unfettered growth: the larger the value, the more rapidly it increases. Yet introduction of the imaginary unit into the function converts it, almost magically, into a sinusoidal function. The cosine and sine, identical except for a phase shift (Figure 6), model the daily, yearly and zodiacal cycles; ripples on the pond; the propagation of sound or light; vibration in all its forms.

Blavatsky commented on the first five digits in the value of π: "The Three, the One, the Four, the One, the Five . . . represent 31415—the numerical hierarchy of the Dhyan-Chohans [celestial beings] of various orders, and of the inner or circumscribed world."[94] The assignment of the letter π to the ubiquitous number was made in 1706, by the Welsh mathematician William Jones. It is the sixteenth letter of the Greek alphabet and has a value of 80 in Greek gematria. The number 314 is the value of Metatron, מטטרון, in the Hebrew gematria; Metatron is the Recording Angel, or "the One behind the throne," mentioned in the *Book of Enoch* and occasionally in the Talmud. The constant π may merit the accolade "transcendental" in more than the narrow mathematical sense.

Esotericist William Eisen included in his book most of the symbols and equations in the section *Mathematical Properties*, above. He referred to π as one of the "two most important ratios in the universe"[95] (the other being the golden ratio, the proportions of the golden rectangle). He then attempted to interpret its value in terms of Hebrew Gematria and the Tarot. Eisen also discussed the constants e and i, providing the infinite series for the former and even describing the complex plane and citing Euler's Identity.[96] He related all four constants, not altogether persuasively, to the geometry of the Great Pyramid of Egypt. Eisen was a pioneer in incorporating higher mathematics into his work, but he contributed little to our understanding of its esoteric significance.

Order and Chaos

Figure 9. Indra slaying the dragon Vritra with a thunderbolt. Khmer Temple Complex, Cambodia.

From ancient times order and chaos have been in a state of mutual tension. This section explores a class of mathematical problems, in which chaos arises from what appears to be perfect order, and contrasts it with mythological depictions of the conquest of primeval chaos by divine order. A surprising, paradoxical outcome is that mathematical chaos can produce greater aesthetic value than the "primeval order" from which it emerges.

Chaos from Order

Chaotic behavior was first encountered by Henri Poincaré at the beginning of the twentieth century—and even by Isaac Newton three centuries earlier—in connection with the "three-body problem."[97] Chaos theory became a field of serious study in the 1960s. It refers to a category of initial-value problems, usually but not necessarily defined by nonlinear differential equations, in which the solution advances in a time-like direction. In contrast to more familiar situations, a chaotic system never settles down to a constant, periodic, or exponentially divergent pattern. Instead, the solution meanders, in a seemingly random fashion, around a set of values known as a *strange attractor*.[98]

The traditional belief—almost a sacred tenet of mathematics—was that, given a system of differential equations for which solutions exist, and given a complete set of boundary conditions, solutions would be calculable and repeatable using alternative calculation methods or different computers. Indeed, calculability and repeatability were considered essential elements of *determinism*.

Chaos theory identifies situations in which alternative calculation methods or different computers may yield similar values only for a finite—often small—number of steps. Thereafter the solutions diverge from one another, exhibiting comparable large-scale patterns of behavior but not reproducing the same values. Small errors or perturbations, caused by numerical rounding or discretization of continuous functions, grow exponentially until they dominate the solution. For example, different solutions might be obtained if an initial value were rounded to seven, rather than eight, decimal places. The hypersensitivity to initial conditions—and subsequent approximations—undermines two of the most cherished beliefs in science: calculability and repeatability, and it forced a reinterpretation of determinism.

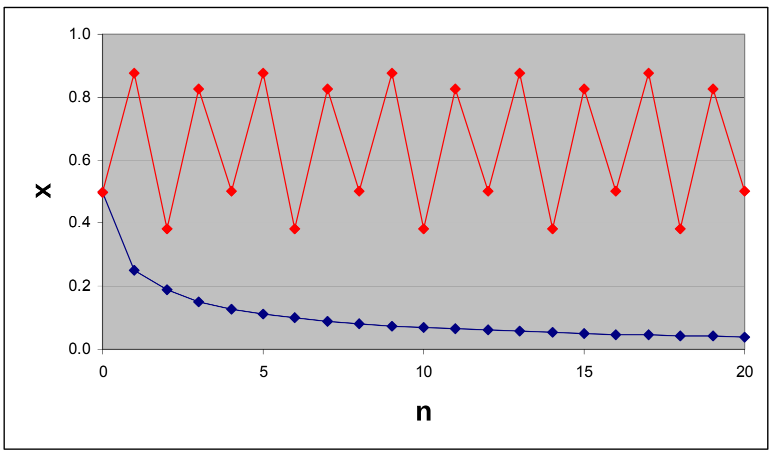
The recurrence equation:

$$x_{n+1} = a\, x_n\, (1 - x_n),$$

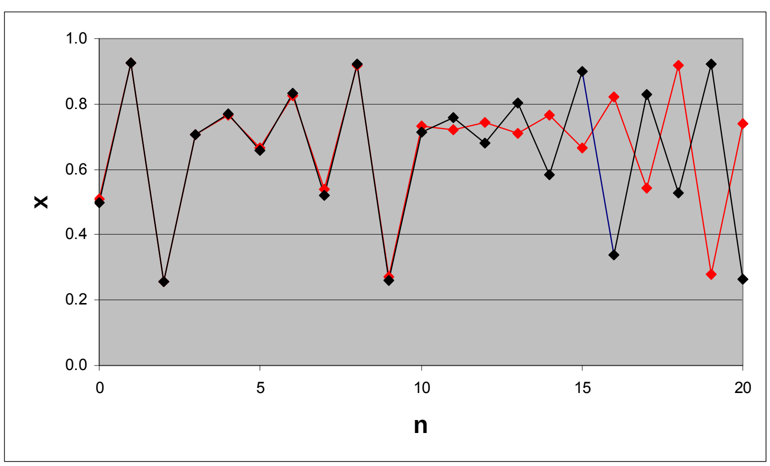
known as the Logistic Map, provides a simple illustration of chaos. Solutions converge to a limit for values of the constant a between 1 and 3, they become periodic for values between 3 and about 3.57, and chaotic for larger values of a. Figure 9 shows calculations for three values of r and an initial value, x_0, of 0.5. In each case, the solution proceeds from left to right.[99] Figure 10(a) shows two well-behaved solutions: a monotonic decay toward zero, when a = 1, and an oscillatory solution with a period of four steps, when a = 3.5. Figure 10(b), where a = 3.7, illustrates chaotic behavior. The blue line is the solution for x_0 = 0.5, and the red line shows the effect of increasing x_0 to 0.51. The two solutions initially remain close together, but then they diverge as the perturbation grows in amplitude; soon the solutions lose any connection with each other. By contrast, a similar perturbation in the calculations shown in Figure 10(a) quickly dies out.

Figure 10. Calculations for the Recurrence Equation $x_{n+1} = a\, x_n\, (1 - x_n)$.

(a) a = 1 (blue), a = 3.5 (red)



(b) a = 3.7

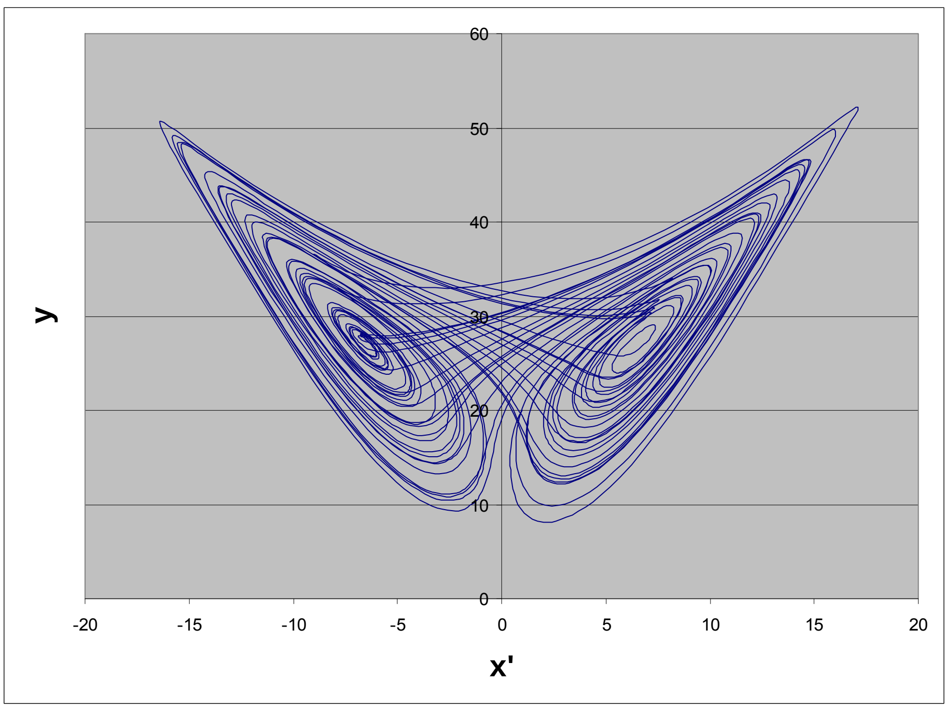


American meteorologist Edward Lorenz observed chaotic behavior in 1963 in a simple three-equation model of thermal convection:

$$\frac{dx}{dt} = \sigma(y - x)$$
$$\frac{dy}{dt} = x(\rho - z) - y$$
$$\frac{dz}{dt} = xy - \beta z$$

The variables x, y and z represent the state of the atmosphere; t is the elapsed time; and σ, ρ and β are empirical constants.[100] Given a set of initial conditions, x_0, y_0, z_0, the equations can be integrated to determine how variables change over time. Over very long periods of time, the values of x and y average out to zero, and z to about 28 (the value of ρ); but over shorter periods they exhibit wide excursions. The trace of the solution in the abstract, "phase," space defined by the coordinates x, y, z forms a strange attractor. A two-dimensional view of the attractor is shown in Figure 11.[101] The intricate—and beautiful pattern has been compared with butterfly wings. Graphs convey some idea of the beauty of the attractor, but greater appreciation can be gained by watching its formation in real time.[102]

Figure 11. The Lorenz Attractor



Two-dimensional views, like the one in Figure 11, might suggest that the trace crosses itself, but in fact, it passes behind or in front of earlier values. The solution never repeats itself. Instead, it gradually fills in the three-dimensional space of the attractor. A prerequisite for chaos is nonlinearity; for instance, Lorenz's equations contain the products xy and xz, while the Logistic Map involves x^2.[103]

In addition to appearing in mathematical equations, chaos is observed in the motion of articulated pendulums, planetary orbits, water dripping from faucets, electrical circuits, chemical reactions, the firing of brain synapses, and—as Lorenz discovered—the weather. Weather forecasts have become more accurate with the development of newer models, but they cannot be made for more than a few days ahead because barometric pressure, humidity, wind patterns, and other factors comprise a chaotic system. Small perturbations, in both initial values and the calculations themselves, quickly become amplified and dominate computed forecasts. What Lorenz called the "butterfly effect" suggests that a butterfly flapping it wings in one part of the world could cause a tornado or hurricane in another.

Importantly, chaotic systems contain bifurcation points at which solutions take on unforeseen properties. An example, was the value of a = 3.57… in the Logistic Map. Solutions on one side of the bifurcation point provide no clue to behavior on the other.

Order from Chaos

The creation stories of multiple cultures described formation of the world from primeval darkness, formlessness, or chaos. In *Genesis* we read: "[T]he earth was without form, and void; and darkness was upon the face of the deep. And the Spirit of God moved upon the face of the waters. And God said, Let there be light: and there was light."[104] The *Zohar*, the primary text of the medieval Kabbalah, described the emergence of color in the void

> A spark of impenetrable darkness flashed within the concealed of the concealed from the head of Infinity—a cluster of vapor forming in formlessness … not white, not black, not red, not green, no color at all… It yielded radiant colors. Deep within the spark gushed a flow, splaying colors below.[105]

According to the Vedas of ancient India, the world was formed from primeval chaos. *Rita*, or Order, was considered to be a divine principle that ensured social stability as well as the regularity of the seasons and night and day.[106] Rita also brought beauty and symmetry to the world; a verse from the *Rig Veda* tells us: "Firm-seated are the foundations of Rita, in its lovely form are many splendid beauties."[107] A modern writer echoes that sentiment: "beauty is an expression of that great law of harmony and equilibrium that gives order and rhythm to the universe."[108]

The preservation of order, in the Vedic system, required the performance of appropriate rituals by the priests and people. It also required attention by the gods. As the champion of Rita, the supreme God Indra was locked in conflict with the dragon Vritra, who represented chaos. The *Rig Veda* exhorts Indra to emerge victorious: "Indra, manliness is thy strength; destroy Vritra, win the waters, acclaiming self-dominion."[109] Indra scored a major victory over the dragon by sending, not one son—as many gods have done—but *twins*, the Asvini, into the world. He eventually killed Vritra with a thunderbolt (Figure 9, at the beginning of this section.)

Ernest McClain's research showed that the story of Indra, Vritra, and the Asvini has a musical correspondence. Long before Pythagoras took up a similar quest, Vedic musicians experimented with musical tones, selecting as sacred certain intervals based on integer ratios of either frequency or the corresponding lengths of the pipes or strings of musical instruments. McClain demonstrated how the intervals of the perfect fourth and fifth can be represented, geometrically by the relationship between Indra and his sons.[110] Those intervals, together with the octave, symbolized the emergence of aesthetic order from the chaos of infinite possibility.

Unfortunately, the fourth, fifth and octave do not provide a usable musical scale, and no satisfactory method was ever devised to identify integer ratios to represent the intervening notes. Eventually, the whole concept of basing scales on rational numbers had to be abandoned. In modern western music, every semitone in the scale is assigned a frequency ratio of $2^{1/12}$.

In the abandonment of Vedic-Pythagorean musical scales (and their derivatives), and in the discovery of mathematical chaos, the divine order, secured by Indra at such great cost, came to naught. Yet music of all genres plays a larger role in people's lives than ever before. Digital processing has allowed music to be recorded and shared at a quality approaching that of live performances.

As for mathematical chaos, numerical methods of solution and computer graphics have allowed us to experience the beauty of strange attractors—geometric forms never previously seen or envisioned. We are seeing the emergence of a new kind of order, one perhaps that only now are we in a position to understand. While specific *quantities* may not be predictable, strange attractors assume the same *patterns* in every relevant calculation.[111] In this instance, and indeed more generally, mathematics is moving from a quantitative to a qualitative emphasis.

Reflections and Synthesis

Mathematics has been a crowning achievements of the fifth root race and the unfoldment of *manas*. It builds on the problem-solving skills of the concrete mind, but it also reaches into higher mental levels, where abstract concepts are formulated and manipulated. Individuals like Euclid, Brahmagupta, Ibn al Banna, Leibniz, Euler, Gauss, Maxwell, Russell, Einstein and Gödel saw reality in new ways. Mathematics is about the study, and occasionally the building, of forms at every level of reality.

Exoteric and Esoteric Mathematics

Nurtured by some of humanity's greatest minds, mathematics models the physical universe—from subatomic levels to galactic superclusters—to a degree that amazes even scientists. Laws of nature, expressible in mathematical terms, seem to be obeyed throughout the universe. Hungarian mathematical physicist Eugene Wigner exclaimed: "It is difficult to avoid the impression that a miracle confronts us here."[112] Yet if the WORD brought the universe into being, James Jeans' assertion that the universe is a mental construct is completely understandable. Mathematics may be part of the language in which the WORD was uttered, while the laws of nature are a manifestation of divine Will.

We must not fall into the trap of assuming that *particular* mathematical constructs have divine sanction. Euclidean geometry, musical scales based on integer frequency ratios, and circular planetary orbits all had to be discarded—sometimes with grave concerns that God would be offended.

Pure mathematics is not constrained by observations of physical reality or by assumptions about the physical world. It long reigned supreme as "queen of the sciences," acclaimed as a self-consistent body of axioms, logical inferences, and theorems whose proofs could forever withstand assault. Although that ideal has lost credibility, pure mathematics continues to expand and grow.

Mathematics, and indeed all the sciences, are driven by the search for Truth. And, while that Truth may be narrowly defined, it is based on systems of verification that transcend individual or group prejudice. Alone among major fields of endeavor, it has remained immune to political, religious and economic pressures—not necessarily in its applications but certainly in its methods. No totalitarian regime, religious sect, or industrial oligarchy has been able to subvert the formulation or proof of mathematical theorems or to sabotage the norms accepted by the mathematical community.[113]

Esotericist Rudolf Steiner (1861–1925) warned of a different threat. He complained about the modern world's obsession with quantification, attributing it to the growing influence of the evil demigod Ahriman: "Nothing can stand up against figures," he complained, "because of the faith that is reposed in them; and Ahriman is only too ready to exploit figures for his purposes."[114] "[F]igures are not a means whereby the essential reality of things can be proved—they are simply a means of deception! Whenever one fails to look beyond figures to the *qualitative*, they can be utterly deceptive."[115]

Perhaps Steiner anticipated the threats posed by Gödel's Incompleteness Theorem or chaos theory; or perhaps he anticipated the use and misuse of statistics to influence public opinion; if the last, he had genuine grounds for concern. The field of statistics is driven by our imprisonment in linear time. It also represents an attempt to argue from the particular to the general, rather than from the general to the particular. Be that as it may, mathematics is not just about quantification; it reaches far beyond numbers. Moreover, we do not know whether Ahriman exists, other than as an archetype.

Even exoteric mathematics, developed for utilitarian purposes or for its intellectual merit, has an aesthetic dimension. Mathematicians frequently comment on the "elegance" of solutions or theorems. Furthermore, the discovery of new proofs or theorems, the construction of new calculation methods, even progress in mathematics courses, can evoke a sense of awe. The sudden breakthrough in a problem that has long perplexed the individual—or the entire mathematical community—is not altogether different from the experience of a mystic encountering the divine presence.

Given such reactions among professional mathematicians of our own time, it is not surprising that a pervasive belief has existed from time immemorial that mathematics could reveal higher truths or values. That belief led people to look for, and in many instances discover, profound aesthetic and esoteric significance in numbers, geometric shapes and symbols. Aware of the tendency of the human mind to see patterns where there may be none, and aware that aesthetics is at least partly personal in its assessments, we must assess claims with caution. Nevertheless, the aesthetic and esoteric dimensions of mathematics have persuasive power that is hard to dismiss.

The implications may be far-reaching. Dorje Jinpa commented: "Once it is realized that the laws which govern art, and the laws which govern physics, and the laws which govern the natural growth of the spirit, are the same laws, we can begin to apply the discoveries made in one field to any other."[116] Perhaps he should have said, "the laws which govern physics and *mathematics*."

The western esoteric literature contains many works on gematria and sacred geometry. In addition to the criteria mentioned earlier, gematria depends for its validity on the belief that letters and words have significance beyond the utilitarian purpose of recording and communicating speech. Ancient cultures sometimes claimed that their languages were of divine origin. Nobody would make such a claim about English, which has drawn upon many earlier tongues and has made numerous concessions to cultural changes. Nonetheless, we have seen the use of English gematria in the trans-Himalayan teachings—and this usage encourages us to pay greater attention to gematria and numerology than we might otherwise feel was warranted.

This article has gone farther than most previous studies by looking for esoteric significance in equations, theorems, and other mathematical constructs. As noted in the Introduction, mathematics need not be understood in detail. Rather, it should be allowed to stimulate the intuition; it should be regarded as a basis for meditation. For example, the infinite series for π can lead the mind toward infinity in somewhat the same way as does a mandala.

The mathematics of the point, the line, and the circle epitomize order, stability, regularity, predictability, the unswerving direction, the endless turning of the Wheel. It calls to mind spiritual paths like the monastic life, with its discipline of labor and the rhythm of meditation and collective prayer. Progress is being made, and many hermits and monks have attained the heights of spirituality. But the work is methodical, and "dark nights of the soul"—which can last months or years—are endured with cheerful patience.

The mathematics of chaotic processes is quite the opposite. Order appears to break down, the future is unpredictable; solutions are organic and exploratory, and wide excursions are possible. Small causes can have large effects; indeed, the amplification of disturbances may underlie the arrow of time and irreversibility mandated by the Second Law of Thermodynamics.[117] Mathematical chaos calls to mind the work of active discipleship, ever encountering new challenges.

Mathematical chaos is not statistical randomness; it is a new kind of order, less obvious but real and meaningful. Chaos theory models evolutionary processes in which new, unexpected, dynamic—and, most importantly, *beautiful*—forms can emerge. Life, in all its stages, from the mineral, to the vegetable, the animal, the human, and the superhuman kingdoms is modeled more accurately by chaotic systems than by "stable" ones. Significantly, chaotic systems are unstable at bifurcation points. Such a point behaves like a teetering rock; a small push can tip the rock one way or another. If, as esotericists affirm, consciousness impinges on the physical plane, it could do so most efficiently at bifurcation points.

Mathematics, the Mysteries, and the Spiritual Path

Mathematics was once viewed as an element of the sacred mysteries. The theorems of Euclidean geometry were shared only with students sworn to secrecy. Geometry is no longer protected in that manner, but mathematics' role in the mysteries continues. Alice Bailey described the secrets revealed at the seven initiations available within the planetary system:

> These seven secrets are simply short formulas, not of mantric value, such as in the case of the Sacred Word, but of a mathematical nature, precisely worded so as to convey the exact intent of the speaker. To the uninitiated they would look and sound like algebraical formulas, except that each is composed (when seen clairvoyantly) of an oval of a specific hue, according to the secret imparted, containing five peculiar hieroglyphics or symbols.[118]

She added: "It will now be apparent why so much stress is laid upon the study of symbols."[119]

As noted earlier, the age-old interest in "sacred mathematics" may hint at its use by the masters during the Atlantean era. Mathematics may be one of the expressions of the initiatory language Sensa. Jinpa declared that Sensa, at its highest level, is an expression of the Divine WORD. As the divine impulse descends into manifestation, it expands into archetypes, thoughtforms, and finally into a myriad of forms at lower levels of consciousness. Jinpa explained: "esoteric symbols represent ideas on a level where the parts, the dualities of opposites, are realized as a synthesis." Perhaps in contemplating mathematical symbols, we can acquire an understanding of Sensa at whatever levels may be accessible to us.

Mathematics is a *universal* language in the full sense of that term. Scientists have long recognized that the mathematical laws of physics are obeyed throughout the universe. Also, Boole, Whitehead and Russell recognized that logical operations expressed in mathematical notation could transcend ordinary linguistic barriers. With appropriate explanations of notation, exoteric mathematics no doubt could be understood by beings of comparable mental ability beyond this planet. Esoteric mathematics, as a sublanguage of Sensa, may also be capable of expressing truths that are valid and comprehensible throughout the universe.

Like all systems of sacred symbolism, mathematics has its priesthood, whose members may seem to outsiders to be secretive custodians of truths veiled to the masses. "The mathematician is . . . regarded as the hermit who knows little of the ways of life outside his cell, who spends his time compounding incredible and incomprehensible theories in a strange, clipped, unintelligible jargon."[120] Yet entry to the priesthood is not arbitrarily restricted. Many more people could participate—or at least take sufficient interest to break down barriers separating mathematics from the larger body of esoteric studies.

Physicist Fritjof Capra suggested that mathematics' isolation may be coming to an end. One reason lies in greater understanding of its goals and ideals. Another—which Galileo would have endorsed—is appreciation of its *necessity*:

> [T]he understanding of pattern is crucial to understanding the living world around us; and . . . all questions of pattern, order, and complexity are essentially mathematical.[121]

Mathematics is an attractive pursuit for individuals on the Third or Fifth Rays, especially in combination: "[T]he third and fifth rays together make the truly great mathematician who soars into heights of abstract thought and calculation."[122] Complementary Seventh Ray influence may enhance practical application, while Fourth Ray influence may enhance an appreciation of mathematics' aesthetics. Every ray presents both opportunities and challenges.

The Third Ray came into manifestation in 1425, and the Fifth Ray in 1775.[123] The great advances in exoteric mathematics that have been made in recent centuries no doubt reflect the influence of those rays and their effect of attracting Third- and Fifth-Ray individuals into incarnation. Clearly, the Planetary Hierarchy intended to stimulate work in exoteric mathematics. There is every reason to believe that the Hierarchy also wanted relevant concepts and methods to be incorporated into esoteric mathematics. This realization makes studies of the Fibonacci series and the golden rectangle, Eisen's pioneering work, and the present work both necessary and urgent. The potential for future research is enormous, and all interested authors are encouraged to contribute.

In the approach to sixth-root-race consciousness, mathematical endeavor can be expected to move increasingly to intuitive levels. Synthesis is already overtaking analysis in importance, even in exoteric mathematics.

With an appreciation of the processes and goals involved, mathematics could open up new avenues for, and draw more people into, building the individual and collective antahkarana. For those ready to go farther, we are reminded of Alice Bailey's advice on meditation for Fifth-Ray disciples, leading up to disintegration of the causal body:

> [I]t is the bending of every mental quality and the controlling of the lower nature so that one supreme endeavor is made to pierce through that which hinders the downflow of the higher knowledge. It involves also the will element (as might be expected) and results in the wresting of the desired information from the source of all knowledge.[124]

Mathematics can provide one component in the expansion of consciousness needed on the initiatory path. The question remains whether a one-pointed pursuit of mathematics is a wise spiritual path. We are warned: "Anyone who over-exalts the concrete mind and permits it continuously to shut out the higher, is in danger of straying on the left-hand path."[125] Yet successful mathematicians use the higher mind to great advantage. And mathematicians in general form one of the least likely groups to be swayed by dangerous emotional forces.

The greater challenge for mathematicians may be to overcome total absorption in their studies. Mystics face a similar challenge. For them, the solution lies in allowing love to flow horizontally as well as vertically. Mathematicians need to broaden their focus, so that they turn outward to the world. Service must become as important as understanding higher reality.

For their part, esotericists could learn much from the mental discipline that mathematicians impose on themselves. It includes insistence on rigor in the search for truth. Buddhist teachings insist:

> [W]e must not believe in a thing said merely because it is said; nor traditions because they have been handed down from antiquity; nor rumors, as such; nor writings by sages, because sages wrote them . . . . But we are to believe when the writing, doctrine, or saying is corroborated by our own reason and consciousness.[126]

Following the example of pure mathematics, esotericists should work from the universal to the particular and from the general to the specific. And they should understand that their quest is one of discovery more than innovation. Both mathematics and esotericism aim to discover relationships that have always existed.[127]

Conclusions

This article was written in the belief that mathematics has an esoteric dimension, as well as its primary goal of exploring the reaches of the concrete and abstract mind, and its long-recognized aesthetic value. The article reviewed the development of exoteric mathematics, drawing attention to its enormous power and wide applicability, but also recognizing intrinsic limitations that have come to light since the early twentieth century.

The article briefly examined gematria and the "traditional" sacred geometry addressed in the literature. The motivation to discuss gematria stemmed largely from its appearance in the trans-Himalayan teachings. The vesica piscis was chosen to illustrate traditional sacred geometry because of intrinsic interest and also because of its connections to the material that followed. The geometry of the golden rectangle, its appearance in nature, and its relationship to the Fibonacci series, have not been discussed herein since they have been covered in great detail elsewhere. Nor has space permitted a discussion of the geometries believed to be encoded in architecture and the landscape.

The geometric symbols of the point, the line, and the circle were examined in considerable depth, from both a mathematical and an esoteric standpoint. The relevant mathematics has been explored in greater detail than previous works on sacred geometry have done. The article compared and contrasted the representation of the circle in Cartesian coordinates and in the complex plane, and the discussion extended to the three enigmatic numbers: π, e and i that feature therein.

The esoteric symbolism of the point, line, and circle has been recognized since the dawn of time, and it is discussed in the works of Blavatsky and Bailey. The line, depending on orientation, can symbolize either Love or Will. The horizontal and vertical lines can be related to the paths of discipleship and initiation, and their juxtaposition forms the cross, with special relevance to the fourth initiation. The point within the circle has rich symbolism, encompassing the Great Invocation as well as the expression of Logoic essence within its ring-pass-not.

Even with its use of analytical geometry, this article may have only scratched the surface of esoteric mathematics. The point, line and circle have still been considered within the framework of Euclidean geometry—and they are shapes that can be constructed with ruler and compass. To assert that other shapes or other geometries are "less sacred" betrays a limited perspective. A larger field of sacred geometry waits to be explored, involving more complicated shapes and other geometries. Many people already meditate on the intriguing and beautiful fractal image known as the Mandelbrot set—which, incidentally, resides in the complex plane.[128]

Chaos theory, which threatens the time-honored trust in scientific determinism—even causality—was illustrated by two examples. A very simple one demonstrated the emergence of chaos, and the hyper-sensitivity to initial conditions that makes long-term predictions impossible. The second involved the integration of coupled differential equations to demonstrate the geometry and beauty of a strange attractor. Chaos, in the mathematical sense, may represent the breakdown of order. But from a larger perspective, it may imply the emergence of new kinds of order, ones that model the evolution of life at all its levels.

Given the observation that the physical universe is the manifestation of Thought, and mathematics' possible associations with Sensa, the language of high initiates, more esotericists should be encouraged to study mathematics. As they do so, they could become receptive to teachings that would be difficult or impossible for Higher Intelligences—on this planet or elsewhere—to communicate by other means. Correspondingly rich opportunities exist for mathematicians to move toward esotericism by contemplating the larger implications of their work; many may already have done so but lack the terminology and philosophical framework to express their insights.

During the fifth root race mathematics has primarily been a quest for Truth at the mental level—both concrete and abstract. As we approach the emergence of the sixth root race, it will likely expand to the intuitive level.

The limitations of exoteric mathematics and its applications are already coming to the surface. Its utilitarian value has never been greater, but the fundamental premises on which mathematics rests no longer go unquestioned. Certainly, we cannot yet speak of the rise and fall of mathematics, but the cracks in the walls of the once mighty edifice point in the direction of evolving human consciousness. The outer form is being weakened to allow the indwelling life to expand and express itself in new ways. As time goes on we can expect greater emphasis to be placed on the aesthetic, intuitive and esoteric dimensions of mathematics. A group effort will be most successful, and hopefully this article will stimulate interest and encourage work on a broad front.

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1 Bertrand Russell, letter to Gilbert Murray, April 3, 1902. Online: http://www.notable-quotes.com/r/russell_bertrand_iv.html#UvOiqPUtcyHGP3ps.99. (Last accessed Nov. 11, 2015).

2 Marcus P. F. du Sautoy, *A Brief History of Mathematics*, part 1. Online: http://www.bbc.co.uk/programmes/b00sr3fm. (Last accessed Aug. 1, 2015).

3 Alice A. Bailey, *Initiation, Human and Solar*, New York: Lucis, 1922, 224.

4 Alice A. Bailey, *A Treatise on White Magic*, New York: Lucis, 1934, 379.

5 Dorje Jinpa, *Sensa: The Lost Language of the Ancient Mysteries* (Ashland, OR: Pentabarba, 2012), 168.

6 Ibid., 169.

7 Except for the use of graphical methods and animated simulations—and except for Eisen's work mentioned herein—the sacred mathematics in the literature is based on elementary arithmetic and the geometry of the straight-edge and compass.

8 Alice A. Bailey, *Letters on Occult Meditation* (New York, NY: Lucis, 1922), 5.

9 Alice A. Bailey, *Esoteric Psychology* I (New York, NY: Lucis, 1936), 159.

10 Ibid., 163.

11 Where the relevant tools are covered in educational curricula varies from place to place. The author learned all the methods of analytical geometry used here in high school. Chaos theory had not yet been discovered by the time the author completed college. Knowledge of numerical methods for integrating partial differential equations was acquired "on the job."

12 William Eisen, *The Universal Language of Cabalah* (Marina del Rey, CA: DeVorss & Co.1989).

13 People who would be loath to admit incontinence or sexual impotence eagerly boast ignorance of mathematics—and are applauded for doing so.

14 Jagadguru Shankaracharya, *Vedic Mathematics* (New York, NY: Samuel Weiser, 1965).

15 Plato, *Timaeus* 31C-32C. Desmond Lee (ed.), *Timaeus and Critias* (London: Penguin Classics, 1965), 31C.

16 The Arrow Paradox, one of several posed by the Greek philosopher Zeno, claimed that motion is illusory, because an arrow appears to be stationary during infinitesimal intervals of time. Newton and Leibniz laid the foundations of the differential calculus by showing that the *ratio* of distance traveled to the corresponding time interval remains finite as the latter tends to zero.

17 In the age of Scholasticism, whose most famous exponent was Thomas Aquinas (1225–1274), theology wore that mantle.

18 Deriving the arithmetic expression for $x \vee y$ is left as an exercise for the student.

19 In classical mathematics, tautological axioms like "the whole is greater than the part" were distinguished from *postulates*, which were believed to be self-evident from experience. For example, Euclid's first postulate was: "It is possible to draw a straight line from any point to any other point." Notions of self-evidence have since been discarded on the grounds that they limit a theorem's range of application, and no distinction is now made between axioms and postulates.

20 Tensors are arrays of variables or functions that can be manipulated as an algebraic unit, subject to applicable rules. For example, the tensor G, which describes the curvature of four-dimensional space-time, is an array of $4 \times 4 \times 4 \times 4 = 256$ elements. Fortunately, from a computational standpoint, only 20 are mathematically independent of one another.

21 Schrödinger's cat, in the famous thought experiment, is simultaneously alive and dead.

22 Measurement and observation imply the involvement of human consciousness.

23 Some theoretical physicists claim that properties of Schrödinger's Wave Equation can be derived from axioms of symmetry alone, without reference to empirical data.

24 James Jeans, *The Mysterious Universe* rev. ed. (London, UK: Cambridge Univ. Press, 1930/1948), 139. Most scientists have simply ignored Jeans' assertion, few have tried to refute it, and some have endorsed it. See for instance: Richard C. Henry, "The Mental Universe," *Nature* (vol. 436, no. 7, July 2005), 29.

25 Harriette A. Curtiss & F. Homer Curtiss, *The Key to the Universe* (Albuquerque, NM: Sun Books, 1917), 17.

26 Inconsistency may be obvious in a small numbers of simple axioms, for example: $A > B$, $B > C$, $C > A$. But it may not be apparent in large, complicated axiomatic systems.

27 These criticisms are not addressed to statistical mechanics, which has played a central role in fields like thermodynamics.

28 For a good overview of traditional sacred mathematics, see John Michell, *The Dimensions of Paradise* (San Francisco: Harper & Row, 1971).

29 The canonical Hebrew Bible was written in Hebrew. The *Septuagint*, published in Alexandria for use by Jews of the Diaspora, consisted of Greek translations of the Hebrew Bible and additional books known as the *Apocrypha*. The whole of the New Testament was written in Greek.

30 It may be noted that the numbers 6, 90 and 900 are missing from Table 1(b). Letters once occupied those positions, but they had fallen into disuse by the time of classical Greece. Clearly, the letter–number equivalences were decided at a very early period in Greek history.

31 Note that Hebrew is written from right to left. The numerical values are given in the order of their English transliterations (left to right).

32 Latin and English do not use letters as numerical symbols, and different rules have emerged for assigning numbers to the letters of the respective alphabets. Rules must also be devised for dealing with archaic letters no longer in general use, or with new letters, like k, imported from Greek after the Latin alphabet took on its classical form.

33 Frederick B. Bond and Thomas S. Lea, *Gematria: A Preliminary Investigation of The Cabala Contained in The Coptic Gnostic Books* (London, UK: Thorsons, 1917), especially appendix C. One such phrase is "Son of Man," Υιος του Ανθρωπου, with a value of 2,960, or 80 × 37—or twice the value of Χριστος.

34 See for example, Gary L. Cobb, *Three Religions, One Temple Mount* (Maitland, FL: Xulon Press, 2007), 252.

35 Alice A. Bailey, *A Treatise on Cosmic Fire*, (New York, NY: Lucis, 1925), 306.

36 Alice A. Bailey, *Esoteric Astrology* (New York, NY: Lucis, 1951), 427.

37 *Revelation* 13:18.

38 Aside from being divisible by 37, the number 666 is the grand total of the numbers in the 6 × 6 Magic Square of the Sun, in which each column, row and diagonal sums to 111. This value, the sum of the numbers on the perimeter, the sum of the four central elements, and 666 itself are all divisible by 37. It is worth noting that esotericists Rudolf Steiner and Max Heindel claimed that Christ was a spirit of the sun.

39 The RAN code contrasts with the Alpha Number (AN) code, which assigns the numbers 1 through 26 to the alphabet, without repetitions. The RAN and AN codes both reduce to the same root values. For more on the English gematria see William Eisen, *The English Cabalah* (Camarillo, CA: DeVorss, 1980).

40 Bailey, *Esoteric Psychology* I, 346. Capitalization in original. Note that on p. 155 of the same book, Bailey identified 22 as the number of the adept.

41 Alice A. Bailey, *The Rays and the Initiations* (New York, NY: Lucis, 1960), 79. Capitalization in original.

42 John Berges, *Sacred Vessel of the Mysteries* (Northfield, NJ: Planetwork Press, 1997); also *Hidden Foundations of the Great Invocation* (Planetwork Press, 2000). For information on the Great Invocation see Alice A. Bailey, *The Externalization of the Hierarchy* (New York, NY: Lucis, 1957), 488ff.

43 Bailey, *The Rays and the Initiations*, 80.

44 Ibid.

45 See for example Curtiss & Curtiss, *The Key to the Universe* and *The Key of Destiny*.

46 See for example Bailey, *Initiation, Human and Solar*, 4.

47 For a discussion of ley lines see for example Paul Screeton, *Quicksilver Heritage* (Wellingborough, UK: Thorsons, 1974).

48 For a brief introduction to this topic, see the website: https://www.mathsisfun.com/numbers/golden-ratio.html. (Last accessed Aug. 19, 2015).

49 Michell associates the left-hand circle with matter, and the right-hand one with spirit; see Michell, *The Dimensions of Paradise*. Mayananda offers the opposite association; see Mayananda, *The Tarot for Today* (Zeus, Easton, Pa: 1963), 139.

50 Mayananda, ibid., 138-139.

51 Plato, *Timeus* (trans: Benjamin Jowett), Internet Classics Archive, §35A. For a discussion of Plato's writings on the soul, see John F. Nash, "Plato: A Forerunner," The Beacon, July/Aug. 2004, 18-24.

52 Bailey, *A Treatise on White Magic*, 35.

53 See for example Michell, *The Dimensions of Paradise*, 70.

54 Irrational numbers will be discussed in more detail later.

55 William Stirling, *The Canon* (London: Elkin Matthews 1897), 108, 136-137. A better approximation can be obtained by reducing the

value of Zeus to 611, and a tolerance of ±1 is often accepted in gematria.

56 David Fiedler, *Jesus Christ, Sun of God* (Wheaton, IL: Quest, 1993), 72.

57 David P. Myers & David S. Percy, *Two-Thirds,* London: Aulis, 1993, appendix.

58 Michael S. Schneider, *A Beginner's Guide to Constructing the Universe* (San Francisco: Harper Collins, 1994), 37.

59 For a readable account of Cantor's work on transfinite numbers see Edward Kasner & James Newman, *Mathematics and the Imagination* (New York, NY: Penguin Books, 1940/1968), 48ff. A set of *countable* numbers can be "lined up" in parallel with the natural numbers 1, 2, 3, …. No such relationship can be established with *uncountable* numbers.

60 A polynomial equation is an equation of the form: $a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots = 0$.

61 Eisen mistakenly classified i and the golden ratio as transcendental. See *The Universal Language of Cabalah*, 160.

62 One radian, equal to approximately 57.3 degrees, is the angle subtended by an arc equal in length to the radius of the circle.

63 Note that $\cos(0) = \cos(2\pi) = 1$; $\sin(0) = \sin(2\pi) = 0$; $\cos(\pi/2) = 0$; $\sin(\pi/2) = 1$; $\cos(\pi) = -1$; $\sin(\pi) = 0$; $\cos(3\pi/2) = 0$; $\sin(3\pi/2) = -1$.

64 Several mathematicians contributed to development of the complex plane, notably Gauss.

65 A line in the complex plane is represented by equating the real part of a complex expression to zero: $\mathrm{Re}\{(a + i)(x + iy) + b\} = 0$.

66 One way to demonstrate the equivalence is by expanding the cosine, sine, and exponential functions as Taylor series and comparing corresponding terms.

67 See for example http://www.livescience.com/51399-eulers-identity.html. (Last accesses Aug. 24, 2015).

68 *1 Kings* 7:23-26; *2 Chronicles* 4:2-5.

69 The theoretical impossibility of squaring the circle was proved in 1882. Notwithstanding, the Indiana House of Representatives passed a bill, fifteen years later, that would have incorporated such a method into state law. The method yielded a value of $\pi = 3.2$. Fortunately, the bill died in the state Senate.

70 Unfortunately, the ancients lacked accurate methods of calculating square roots.

71 This series, developed by Simon Plouffe and included in a 1995 paper by David H. Bailey, Peter Borwein, and Plouffe, is not only highly efficient; it also has the property that individual terms can be calculated in the hexadecimal (base-16) expansion for π.

72 For example, the decimal expansion of 22/7—often used as an approximation to π—is 3.142857142857…, which converges to endless repetition of the digits 142857.

73 For example $\int dx/\sqrt{(1 - x^2)}$, over the range $-1 < x < 1$, can be evaluated using the substitution $x = \sin\theta$, and yields the value π.

74 According to the American mode of writing dates, March 14, 2015 was 3-14-15. A special celebration was observed at 9:26:54(or 5) in the morning.

75 Bailey, *Esoteric Astrology*, 111.

76 Ibid., 479.

77 Plotinus, *Sixth Ennead*, 6th Tractate. See also Karen Armstrong, *A History of God* (New York: Ballentine, 1993), 102-3.

78 Helena P. Blavatsky, *The Secret Doctrine* I (New York, NY: Theosophical Publishing Co., 1888), 1. Capitalization in original.

79 Alice A. Bailey, *Discipleship in the New Age* I (New York, NY: Lucis, 1944), 335.

80 Bailey, *The Externalization of the Hierarchy*, 488.

81 Blavatsky, *The Secret Doctrine* I, 1. Capitalization in original.

82 Blavatsky, *The Secret Doctrine* I, 4

83 Bailey, *Initiation, Human and Solar*, 165-166.

84 Bailey, *A Treatise on Cosmic Fire*, 159-160.

85 Bailey, *The Rays and the Initiations*, 182.

86 Bailey, *Letters on Occult Meditation*, 31.

87 *Ibid*.

88 Alice A. Bailey, *Esoteric Psychology* II (New York: Lucis, 1942), 352.

89 Ibid., 372.

90 Bailey, *The Rays and the Initiations*, 479.

91 Ibid.

92 Kasner & Newman, *Mathematics and the Imagination*, 87; Fritjof Capra, *The Web of Life* (New York, NY: Anchor, 1996), 143.

93 See for example *Book of Common Prayer* (New York, NY: The Episcopal Church, 1979), 358.

94 Blavatsky, *The Secret Doctrine*, 90.

95 Eisen, *The Universal Language of Cabalah*, 64.

96 Ibid., 162ff.

97 The three-body problem concerns the motion of two planets orbiting a sun. Newton recognized that such motion is unstable and concluded that a Deity was needed to return errant planets to their proper places. Poincaré was

the first to encounter a strange attractor, though he did not coin the term.

98 More generally, an *attractor* is a set of values to which a dynamical system tends, as $t \rightarrow \infty$, for a range of initial values.

99 Calculations and graphs by the author.

100 Edward N. Lorenz, "Deterministic Nonperiodic Flow," *Journal of Atmospheric Sciences* (vol. 20, 1963), 130-141.

101 Calculations and graph by the author. The equations were integrated by an explicit, forward-difference method, using Lorenz's own values of the constants: $\sigma = 10$, $\rho = 28$, $\beta = 8/3$. The horizontal axis in the graph is rotated 12° from the true x-axis. Specifically, $x' = x \cos\theta - z \sin\theta$, where $\theta = 12°$.

102 Two videos of the Lorenz Attractor, available at the time of writing, can be found at: https://www.youtube.com/watch?v=FYE4JKAXSfY and https://www.youtube.com/watch?v=dP3qAq9RNLg. (Last accessed Sept. 29, 2015).

103 One consequence of the nonlinearity is that solutions of the relevant equations cannot be expressed as the sum of more elementary solutions.

104 *Genesis* 1:2-3. KJV.

105 *Zohar*, 1 Bereshit A, 1:1 (Stanford, CA: Pritzker), vol. 1.

106 *Rita* is the root of our words "ritual," "rite" and "right."

107 *Rig Veda*, IV. 23.9. Reproduced in Abinash Chandra Bose, *Hymns from the Vedas* (Mumbai, India: Asia Publishing House, 1966), 7.

108 Dorje Jinpa, "Beauty," *The Esoteric Quarterly* (Summer 2015), 77-80.

109 *Rig Veda*, I. 80.3, pp. 10, 175. Bose explained that "win the waters" may be translated as "win the light."

110 Ernest McClain, *The Myth of Invariance* (York Beach, ME: Nicolas Hays, 1976), 24-28. Later in the same work, McClain identifies a number of other Vedic musical correspondences, including an explanation of the dances of Shiva.

111 See the discussion in Capra, *The Web of Life*, 135-136.

112 Eugene P. Wigner, (1960). "The Unreasonable Effectiveness of Mathematics in the Natural Sciences." Richard Courant lecture in mathematical sciences, New York University, May 11, 1959. Reproduced in *Communications on Pure and Applied Mathematics* (vol. 13, 1960), 1–14.

113 Totalitarianism and sectarianism are regarded as negative characteristics of the Piscean Age. Alice Bailey even attributed the "conquests of science"—and presumably mathematics—to Piscean attitudes. Perhaps these, too, are coming to an end as we move into the Aquarian Age. See her *Education in the New Age* (New York, NY: Lucis, 1954), 3.

114 Rudolf Steiner, lecture, Dornach, Switzerland, November 1, 1919. Reproduced in *The Incarnation of Ahriman* (trans.: M. Barton), Rudolf Steiner Press, 2006.

115 Rudolf Steiner, lecture, November 4, 1919. Reproduced in *The Incarnation of Ahriman*. Emphasis in original.

116 Jinpa, "Beauty."

117 Ilya Prigogine, "The Rediscovery of Time," *The World View of Contemporary Physics* (Albany, NY: State Univ. of New York, 1988). The Second Law of Thermodynamics states that entropy of closed systems can only increase. A glass can shatter, but the shards can never reassemble themselves. Self-organizing systems and biological growth, in which entropy decreases, take place in open systems in which energy is exchanged with the environment.

118 Bailey, *Initiation, Human and Solar*, 165.

119 Ibid.

120 Kasner & Newman, *Mathematics and the Imagination*, 13.

121 Capra, *The Web of Life*, 152-153.

122 Bailey, *Esoteric Psychology* I, 205. See also 204, 212.

123 Ibid., 26, 67. 411. The author is indebted to a reviewer for drawing attention to the relevance of these cycles to the present work.

124 Bailey, *Letters on Occult Meditation*, 17-18.

125 Ibid., 134.

126 *Kalama Sutta of the Anguttaranikayo*. Quoted in Henry Olcott, *A Buddhist Catechism*, part 3, §195, 196. Online: http://www.sacred-texts.com/bud/tbc/tbc09.htm. (Last accessed October 29, 2015).

127 The author is indebted to a reviewer for these suggestions.

128 For images and videos of the Mandelbrot Set see http://www.madore.org/~david/math/mandelbrot.html. (Last accessed Sept. 30, 2015).